Explicit-unit formulations of proposed redefinitions of SI base units

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Abstract

Redefinitions of SI base units have recently been proposed by the CIPM and endorsed, in principle, by the CGPM, based on earlier proposals known as the 'new SI.' The proposed redefinitions of all base units use an *explicit-constant* format involving fixed exact numerical values of seven dimensionally independent reference quantities. In this paper the proposed redefinitions of the seven base units are interpreted in equivalent *explicit-unit* format: each *base-unit* definition is rewritten so that the base unit itself is expressed explicitly as an exact numerical factor multiplying a characteristic quantity formed from the reference quantities, *without introducing other (base or derived) units*. To give some perspective, for comparison in each case, the current SI base-unit definitions, in addition to being listed in their original form, are given in both explicit-constant and explicit-unit formats, as well.

Keywords: explicit-unit redefinitions; new SI; SI base units; redefined SI units.

Running title: Explicit-unit form of proposed SI units

1. Reference quantities

The CGPM has recently endorsed sweeping changes to the definitions of the SI base units [1] based on earlier proposals by five authors known as the 'New SI' [2], and subsequently formally proposed by the CIPM [3]. In the proposed changes, each base-unit definition would be written in an explicit-constant format by fixing the numerical value of one of seven dimensionally independent reference quantities when expressed in terms of the redefined base units [4]. Although it has been claimed that explicit-*unit* definitions have also been given [5], these are actually explicit expressions for the *derived units*: hertz, metre per second, joule second, coulomb, joule per kelvin, reciprocal mole and lumen per watt; i.e., not the *base* units. Therefore, in this paper I will rephrase the recommended explicit-constant definitions of the *base* units themselves in explicit-*unit* format. Each base unit is written explicitly as an exact numerical factor multiplying a characteristic quantity (with the same dimension as the respective base unit) formed from the reference constants—*without introducing any other (base or derived) units*. For perspective, the current SI base unit definitions are given in their original form [6] as well as being restated in explicit-constant and explicit-unit formats.

As is the case for the current definitions, the CIPM-proposed redefinitions of the seven base units are based on a set of seven chosen dimensionally independent reference quantities, the numerical values of which are 'fixed' (i.e., equal to an exact number) when they are expressed in terms of the relevant base units. These are considered to be reference constants [5, 7]. In the current set of SI unit definitions, some of the 'constants' are clearly not constant physically, the classic example being the kilogram artifact, the mass of which is known to be changing due to contamination and handling—although, by definition, it is always equal to *exactly* one kilogram; i.e., the magnitude of the SI base unit for mass is drifting (very slightly) over time. Since the current definitions of the ampere, mole and candela also involve the kilogram, these units are also changing. The instability of the kilogram is one of the motivations for redefining this unit, in particular. A similar situation occurs with the kelvin, currently based on the triple-point of water, which is notoriously sensitive to impurities and other factors. Other proposed unit redefinitions are designed to assign exact values to physical quantities that, according to theoretical physics, are regarded as being fundamental constants, such as the vacuum speed of light, the Planck constant and the elementary charge.

For ease of identification, relevant reference quantities are shown here in bold type where they are used in respective definitions. In the following, $\boldsymbol{\nu}_{\mathbf{Cs}}$ is the frequency of radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom; $\boldsymbol{c}_{\mathbf{0}}$ is the vacuum speed of electromagnetic propagation; $\boldsymbol{m}(\boldsymbol{\mathcal{K}})$ is the mass of the international prototype of the kilogram; $\boldsymbol{\mu}_{\mathbf{0}}$ is the magnetic constant; $\boldsymbol{T}_{\mathbf{TPW}}$ is the temperature of the triple-point of water; $\boldsymbol{M}(^{\mathbf{12}}\mathbf{C})$ is the amount-specific mass of carbon 12; $\boldsymbol{K}_{\mathbf{cd}}$ is the spectral luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz; $\boldsymbol{h}$ is the Planck constant; $\boldsymbol{e}$ is the elementary charge; $\boldsymbol{k}$ is the Boltzmann constant; and $\boldsymbol{N}_{\mathbf{A}}$ is the Avogadro constant.

2. Current and CIPM-proposed definitions

In the following, the order of definitions follows that of the CIPM-proposed redefinitions: second, metre, kilogram, ampere, kelvin, mole and candela. For definiteness, the current CODATA 2010 estimated numerical values of the constants [8] are shown here, with the understanding that the most precise and consistent set of values available would be used at the time of transition, in order to preserve seamless continuity with current definitions.

2.1 Unit of time: second

- Current definition:

The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

- Current definition in explicit-*constant* format:

$$\boldsymbol{\nu}_{\mathbf{Cs}} \equiv 9\,192\,631\,770 \text{ Hz}.$$

- Current definition in explicit-*unit* format:

$$\mathrm{s} \equiv 9\,192\,631\,770\,(1/\boldsymbol{\nu}_{\mathbf{Cs}}).$$

- Characteristic time interval:

$$t^* = 1/\nu_{\mathrm{Cs}} \approx 1.087\ 827\ 757 \times 10^{-10}\ \mathrm{s}.$$

- CIPM-proposed (explicit-*constant*) definition:

The second, s, is the unit of time; its magnitude is set by fixing the numerical value of the ground-state hyperfine splitting frequency of the caesium 133 atom, at rest and at a temperature of 0 K, to be equal to exactly 9 192 631 770 when it is expressed in the unit s^{-1}, which is equal to Hz; i.e.,

$$\nu_{\mathrm{Cs}} \equiv 9\ 192\ 631\ 770\ \mathrm{Hz}.$$

- Equivalent definition in explicit-*unit* format:

The second, s, is the unit of time, equal to exactly 9 192 631 770 ($1/\nu_{\mathrm{Cs}}$), where ν_{Cs} is the ground-state hyperfine splitting frequency of the caesium 133 atom; i.e.,

$$\mathrm{s} \equiv 9\ 192\ 631\ 770\ (1/\nu_{\mathrm{Cs}}).$$

- Characteristic time interval:

$$t^* = 1/\nu_{\mathrm{Cs}} \approx 1.087\ 827\ 757 \times 10^{-10}\ \mathrm{s}.$$

- Comments:

The CIPM-proposed definition is simply an explicit-constant rewording of the current definition. One problem with the CIPM-proposed wording is that it involves another unit (the kelvin) redefined later in the list; and, since the redefined kelvin also involves the second (as well as the metre and kilogram), there is considerable inherent cross-coupling of these definitions. The simplest solution is to refer to the ground state of the caesium atom (as in the current definition)—and omit 'at rest and at a temperature of 0 K.' The architects of the 'New SI' proposals have recently suggested an alternative wording that does not introduce the kelvin, replacing 'at a temperature of 0 K' by 'at a zero thermodynamic temperature' [5]. The wording quoted by Taylor [7] simply uses the 'ground-state' designation without reference to temperature.

2.2 Unit of length: metre

- Current definition:

The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.

- Current definition in explicit-*constant* format:

$$\boldsymbol{c_0} \equiv 299\,792\,458 \text{ m s}^{-1}.$$

- Current definition in explicit-*unit* format:

$$\text{m} \equiv (9\,192\,631\,770/299\,792\,458)\,(\boldsymbol{c_0}/\boldsymbol{\nu_{\mathrm{Cs}}})$$

$$\approx 30.663\,318\,99\,(\boldsymbol{c_0}/\boldsymbol{\nu_{\mathrm{Cs}}}).$$

- Characteristic length:

$$l^* = \boldsymbol{c_0}/\boldsymbol{\nu_{\mathrm{Cs}}} \approx 3.261\,225\,572 \times 10^{-2} \text{ m}.$$

- CIPM-proposed (explicit-*constant*) definition:

The metre, m, is the unit of length; its magnitude is set by fixing the numerical value of the speed of light in vacuum to be equal to exactly 299 792 458 when it is expressed in the unit m s^{-1}; i.e.,

$$\boldsymbol{c_0} \equiv 299\,792\,458 \text{ m s}^{-1}.$$

- Equivalent definition in explicit-*unit* format:

The metre, m, is the unit of length, equal to exactly (9 192 631 770/299 792 458)($\boldsymbol{c_0}/\boldsymbol{\nu_{\mathrm{Cs}}}$), where $\boldsymbol{c_0}$ is the vacuum speed of light and $\boldsymbol{\nu_{\mathrm{Cs}}}$ is the ground-state hyperfine splitting frequency of the caesium 133 atom; i.e.,

$$\text{m} \equiv (9\,192\,631\,770/299\,792\,458)\,(\boldsymbol{c_0}/\boldsymbol{\nu_{\mathrm{Cs}}})$$

$$\approx 30.663\,318\,99\,(\boldsymbol{c_0}/\boldsymbol{\nu_{\mathrm{Cs}}}).$$

- Characteristic length:

$$l^* = \boldsymbol{c_0}/\boldsymbol{\nu_{\mathrm{Cs}}} \approx 3.261\ 225\ 572 \times 10^{-2}\ \mathrm{m}.$$

- Comments:

Again, this is a rewording of the current definition, based on the same characteristic length consisting of the quotient of the vacuum light speed and the caesium frequency. Since the current (and reworded CIPM-proposed) definition of the metre involves the time-unit reference quantity, $\boldsymbol{\nu_{\mathrm{Cs}}}$, as well as $\boldsymbol{c_0}$, any future change in the definition of the second would thus propagate into the explicit-unit definition of the metre.

2.3 Unit of mass: kilogram

- Current definition:

The kilogram is the unit of mass; it is equal to the international prototype of the kilogram.

- Current definition in explicit-*constant* format:

The kilogram, kg, is the unit of mass; its magnitude is set by fixing the numerical value of the mass of the international prototype of the kilogram to be equal to exactly 1 when it is expressed in the unit kg, i.e.,

$$\boldsymbol{m(\mathcal{K})} \equiv 1\ \mathrm{kg}.$$

- Current definition in explicit-*unit* format:

The kilogram, kg, is the unit of mass, equal to exactly $\boldsymbol{m(\mathcal{K})}$, where $\boldsymbol{m(\mathcal{K})}$ is the mass of the international prototype of the kilogram; i.e.,

$$\mathrm{kg} \equiv \boldsymbol{m(\mathcal{K})}.$$

- Characteristic mass for the current definition:

$$m^* = \boldsymbol{m(\mathcal{K})} \equiv 1\ \mathrm{kg}.$$

- CIPM-proposed (explicit-*constant*) definition:

The kilogram, kg, is the unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be equal to exactly 6.626 069 57 × 10^{-34} when it is expressed in the unit $s^{-1}\,m^2$ kg, which is equal to J s; i.e.,

$$\boldsymbol{h} \equiv 6.626\ 069\ 57 \times 10^{-34}\ \text{J s}.$$

- Equivalent definition in explicit-*unit* format:

The kilogram, kg, is the unit of mass, equal to exactly [299 792 458^2/(6.626 069 57 × 10^{-34} × 9 192 631 770)] $(\boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}/\boldsymbol{c}_{\mathbf{0}}^{\mathbf{2}})$, where $\boldsymbol{h}$ is the Planck constant, $\boldsymbol{\nu}_{\mathbf{Cs}}$ is the ground-state hyperfine splitting frequency of the caesium 133 atom and $\boldsymbol{c}_{\mathbf{0}}$ is the vacuum speed of light; i.e.,

$$\text{kg} \equiv [299\ 792\ 458^2/(6.626\ 069\ 57 \times 10^{-34} \times 9\ 192\ 631\ 770)]\,(\boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}/\boldsymbol{c}_{\mathbf{0}}^{\mathbf{2}})$$
$$\approx 1.475\ 521\ 529 \times 10^{40}\,(\boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}/\boldsymbol{c}_{\mathbf{0}}^{\mathbf{2}}).$$

- Characteristic mass for the CIPM-proposed definition:

$$m^* = \boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}/\boldsymbol{c}_{\mathbf{0}}^{\mathbf{2}} \approx 6.777\ 264\ 719 \times 10^{-41}\ \text{kg}.$$

- Comments:

The CIPM-proposed definition of the mass unit, involving the characteristic quantity $m^* = \boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}/\boldsymbol{c}_{\mathbf{0}}^{\mathbf{2}}$, is significantly different from the current definition, which is based on the actual mass of a physical artifact, $\boldsymbol{m}(\mathcal{K})$. Clearly the fixed-Planck-constant definition couples the definition of the kilogram with those of the second and the metre. Although the explicit-unit form of the fixed-$\boldsymbol{h}$ kilogram definition is somewhat easier to comprehend than the explicit-constant form, the characteristic mass is an unphysical extremely small pseudo-mass. In terms of the fixed-$\boldsymbol{h}$ kilogram, the international prototype would no longer have a mass of exactly one kilogram; after transition to the new units, its numerical value and uncertainty would be determined by experiment.

2.4 Unit of electric current: ampere

- Current definition:

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

- Current definition in explicit-*constant* format:

The ampere, A, is the unit of electric current; its magnitude is set by fixing the numerical value of the magnetic constant to be equal to exactly $4\pi \times 10^{-7}$ when it is expressed in the unit s^{-2} kg m A^{-2}, which is equal to N A^{-2}; i.e.,

$$\boldsymbol{\mu_0} \equiv 4\pi \times 10^{-7}\ \mathrm{N\ A^{-2}}.$$

- Current definition in explicit-*unit* format:

The ampere, A, is the unit of electric current, equal to exactly $[(4\pi \times 10^{-7})/(9\,192\,631\,770 \times 299\,792\,458)]^{1/2}[\boldsymbol{m(\mathcal{K})\nu_{\mathrm{Cs}}\, c_0/\mu_0}]^{1/2}$, where $\boldsymbol{m(\mathcal{K})}$ is the mass of the international prototype of the kilogram, $\boldsymbol{\nu_{\mathrm{Cs}}}$ is the ground state hyperfine splitting frequency of the caesium 133 atom, $\boldsymbol{c_0}$ is the vacuum speed of light and $\boldsymbol{\mu_0}$ is the magnetic constant; i.e.,

$$\begin{aligned} \mathrm{A} &\equiv [(4\pi \times 10^{-7})/(9\,192\,631\,770 \times 299\,792\,458)]^{1/2}[\boldsymbol{m(\mathcal{K})\nu_{\mathrm{Cs}}\, c_0/\mu_0}]^{1/2} \\ &\approx 6.752\,656\,351 \times 10^{-13}[\boldsymbol{m(\mathcal{K})\nu_{\mathrm{Cs}}\, c_0/\mu_0}]^{1/2}. \end{aligned}$$

- Characteristic electric current for the current definition:

$$I^* = [\boldsymbol{m(\mathcal{K})\nu_{\mathrm{Cs}}\, c_0/\mu_0}]^{1/2} \approx 1.480\,898\,698 \times 10^{12}\ \mathrm{A}.$$

- CIPM-proposed (explicit-*constant*) definition:

The ampere, A, is the unit of electric current; its magnitude is set by fixing the numerical value of the elementary charge to be equal to exactly $1.602\,176\,565 \times 10^{-19}$ when it is expressed in the unit s A, which is equal to C; i.e.,

$$\boldsymbol{e} \equiv 1.602\ 176\ 565 \times 10^{-19}\ \mathrm{C}.$$

- Equivalent definition in explicit-*unit* format:

The ampere, A, is the unit of electric current, equal to exactly $[1/(1.602\ 176\ 565 \times 10^{-19} \times 9\ 192\ 631\ 770)]\ (\boldsymbol{e}\boldsymbol{\nu}_{\mathbf{Cs}})$, where $\boldsymbol{e}$ is the elementary charge and $\boldsymbol{\nu}_{\mathbf{Cs}}$ is the ground-state hyperfine splitting frequency of the caesium 133 atom; i.e.,

$$\mathrm{A} \equiv [1/(1.602\ 176\ 565 \times 10^{-19} \times 9\ 192\ 631\ 770)]\ (\boldsymbol{e}\boldsymbol{\nu}_{\mathbf{Cs}})$$
$$\approx 6.789\ 687\ 110 \times 10^{8}\ (\boldsymbol{e}\boldsymbol{\nu}_{\mathbf{Cs}}).$$

- Characteristic current for the CIPM-proposed definition:

$$I^* = \boldsymbol{e}\boldsymbol{\nu}_{\mathbf{Cs}} \approx 1.472\ 821\ 919 \times 10^{-9}\ \mathrm{A}.$$

- Comments:

Since many electric currents consist of the flow or oscillation of electrically charged particles, the CIPM-proposed fixed-elementary-charge definition is much more direct and would be more easily comprehended than the current definition. Note, however, the cross-coupling with the definition of the second. With this elementary-charge-based definition, the magnetic constant, μ_0, the electric constant, ε_0, and the vacuum impedance, Z_0, would no longer be exactly known, although $\boldsymbol{c}_{\mathbf{0}} = 1/(\varepsilon_0\mu_0)^{1/2}$ would retain its exact value; in terms of the redefined units, the numerical values and uncertainties of μ_0, ε_0 and Z_0 would be determined by experiment.

2.5 Unit of thermodynamic temperature: kelvin

- Current definition:

The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple-point of water.

- Current definition in explicit-*constant* format:

The kelvin, K, is the unit of thermodynamic temperature; its magnitude is set by fixing the numerical value of the temperature of the triple-point of water to be equal to exactly 273.16 when it is expressed in the unit K; i.e.,

$$\boldsymbol{T}_{\mathbf{TPW}} \equiv 273.16 \text{ K}.$$

- Current definition in explicit-*unit* format:

The kelvin, K, is the unit of thermodynamic temperature, equal to exactly $(1/273.16)\boldsymbol{T}_{\mathbf{TPW}}$, where $\boldsymbol{T}_{\mathbf{TPW}}$ is the thermodynamic temperature of the triple point of water; i.e.,

$$\begin{aligned} \text{K} &\equiv (1/273.16)\, \boldsymbol{T}_{\mathbf{TPW}} \\ &\approx 3.660\,858\,105 \times 10^{-3}\, \boldsymbol{T}_{\mathbf{TPW}}. \end{aligned}$$

- Characteristic temperature for the current definition:

$$T^* = \boldsymbol{T}_{\mathbf{TPW}} \equiv 273.16 \text{ K}.$$

- CIPM-proposed (explicit-*constant*) definition:

The kelvin, K, is the unit of thermodynamic temperature; its magnitude is set by fixing the numerical value of the Boltzmann constant to be equal to exactly $1.380\,6488 \times 10^{-23}$ when it is expressed in the unit $\text{s}^{-2}\text{ m}^2\text{ kg K}^{-1}$, which is equal to J K^{-1}; i.e.,

$$\boldsymbol{k} \equiv 1.380\,6488 \times 10^{-23} \text{ J K}^{-1}.$$

- Equivalent definition in explicit-*unit* format:

The kelvin, K, is the unit of thermodynamic temperature, equal to exactly $[1.380\,6488 \times 10^{-23}/(6.626\,069\,57 \times 10^{-34} \times 9\,192\,631\,770)]\,(\boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}/\boldsymbol{k})$, where $\boldsymbol{h}$ is the Planck constant, $\boldsymbol{\nu}_{\mathbf{Cs}}$ is the ground-state hyperfine splitting frequency of the caesium 133 atom and $\boldsymbol{k}$ is the Boltzmann constant; i.e.,

$$\begin{aligned} \text{K} &\equiv [1.380\,6488 \times 10^{-23}/(6.626\,069\,57 \times 10^{-34} \times 9\,192\,631\,770)]\,(\boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}/\boldsymbol{k}) \\ &\approx 2.266\,665\,135\,(\boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}/\boldsymbol{k}). \end{aligned}$$

- Characteristic temperature for the CIPM-proposed definition:

$$T^* = \boldsymbol{h}\boldsymbol{\nu}_{\mathrm{Cs}}/\boldsymbol{k} \approx 4.411\ 767\ 688 \times 10^{-1}\ \mathrm{K}.$$

- Comments:

Fixing the value of the Boltzmann constant is appropriate because $\boldsymbol{k}$ represents the (invariant) transformation factor between internal energy and temperature. In addition to the Boltzmann constant, the characteristic temperature involves the caesium frequency and the Planck constant. The temperature of the triple point of water would no longer be exact; its numerical value and uncertainty would be determined by experiment.

2.6 Unit of amount of substance: mole

- Current definition:

The mole is the amount of substance of a system that contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is mol. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.
The carbon 12 atoms are understood to be unbound and in their ground state.

- Current definition in explicit-*constant* format:

The mole, mol, is the unit of amount of substance consisting of an aggregate of a specified kind of chemically distinct elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles; its magnitude is set by fixing the numerical value of the amount-specific mass of carbon 12 to be equal to exactly 0.012 when it is expressed in the unit kg mol^{-1}; i.e.,

$$\boldsymbol{M}(^{12}\mathbf{C}) \equiv 0.012\ \mathrm{kg\ mol}^{-1}.$$

- Current definition in explicit-*unit* format:

The mole, mol, is the unit of amount of substance consisting of an aggregate of a specified kind of chemically distinct elementary entity, which may be an atom, molecule, ion, electron, or any other particle or specified group of particles, equal to exactly

$0.012[\boldsymbol{m(\mathcal{K})}/\boldsymbol{M(^{12}C)}]$, where $\boldsymbol{m(\mathcal{K})}$ is the mass of the international prototype of the kilogram and $\boldsymbol{M(^{12}C)}$ is the amount-specific mass of carbon 12; i.e.,

$$\mathrm{mol} \equiv 0.012[\boldsymbol{m(\mathcal{K})}/\boldsymbol{M(^{12}C)}].$$

- Characteristic amount of substance for the current definition:

$$n^* = \boldsymbol{m(\mathcal{K})}/\boldsymbol{M(^{12}C)} \approx 83.333\,333\,33 \text{ mol}.$$

- Comments:

The current definition stems directly from the concept of the mole as an Avogadro number of elementary entities, with the Avogadro number, $\mathcal{N}_{\mathrm{Avo}}$, required to be exactly equal to the gram-to-dalton mass-unit ratio: $\mathcal{N}_{\mathrm{Avo}} \equiv \mathrm{g/Da}$ [9]. With the carbon-12-based dalton given by $\mathrm{Da} = m_{\mathrm{a}}(^{12}\mathrm{C})/12$, this implies that:

$$\mathcal{N}_{\mathrm{Avo}} \equiv \mathrm{g/Da} = (0.001\ \mathrm{kg})/[m_{\mathrm{a}}(^{12}\mathrm{C})/12] = (0.012\ \mathrm{kg})/m_{\mathrm{a}}(^{12}\mathrm{C}).$$

In the formal wording of the current definition, this is interpreted as the number of atoms in 0.012 kilogram (i.e., 12 grams) of carbon 12—but it is fundamentally a mass-unit ratio. Although g/Da is a well-defined number, its actual value is not known exactly.

The amount-specific mass of any entity is, by definition, equal to the entity mass per entity; in particular, for the carbon-12 atom: $\boldsymbol{M(^{12}C)} = m_{\mathrm{a}}(^{12}\mathrm{C})\ \mathrm{ent}^{-1}$, where one entity (symbol ent) is taken to be the atomic-scale unit of amount of substance: the amount of substance consisting of a single entity of that substance—i.e., the (existence of the) entity itself [10]. According to the definition of $\mathcal{N}_{\mathrm{Avo}}$, the mole can be interpreted directly as $\mathcal{N}_{\mathrm{Avo}}$ entities:

$$\mathrm{mol} = \mathcal{N}_{\mathrm{Avo}}\ \mathrm{ent} = (\mathrm{g/Da})\ \mathrm{ent}, \text{ exactly,}$$

i.e.,

$$\mathrm{mol} \;=\; (0.012\ \mathrm{kg})/[m_\mathrm{a}(^{12}\mathrm{C})]\ \mathrm{ent}^{-1}],\ \text{exactly}.$$

This is the origin of the current definition in explicit-unit form:

$$\mathrm{mol} \;\equiv\; 0.012[\boldsymbol{m(\mathcal{K})}/\boldsymbol{M(^{12}\mathrm{C})}].$$

Note that the Avogadro *constant*, defined as the amount-specific number of entities, i.e., $N_\mathrm{A} = N(\mathrm{X})/n(\mathrm{X})$, independent of the kind of entity, X, can be written as:

$$N_\mathrm{A} \;=\; 1\ \mathrm{ent}^{-1} \;=\; \mathcal{N}_\mathrm{Avo}\ \mathrm{mol}^{-1} \;=\; (\mathrm{g/Da})\ \mathrm{mol}^{-1} \;=\; [(0.012\ \mathrm{kg})/m_\mathrm{a}(^{12}\mathrm{C})]\ \mathrm{mol}^{-1}.$$

- CIPM-proposed (explicit-*constant*) definition:

The mole, mol, is the unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles; its magnitude is set by fixing the numerical value of the Avogadro constant to be equal to exactly $6.022\,141\,29 \times 10^{23}$ when it is expressed in the unit mol^{-1}; i.e.,

$$\boldsymbol{N_\mathrm{A}} \;\equiv\; 6.022\,141\,29 \times 10^{23}\ \mathrm{mol}^{-1}$$

- Equivalent definition in explicit-*unit* format:

The mole, mol, is the unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles, equal to exactly $6.022\,141\,29 \times 10^{23}$ $(1/\boldsymbol{N_\mathrm{A}})$, where $\boldsymbol{N_\mathrm{A}}$ is the Avogadro constant; i.e.,

$$\mathrm{mol} \;\equiv\; 6.022\,141\,29 \times 10^{23}\ (1/\boldsymbol{N_\mathrm{A}}).$$

- Characteristic amount for the CIPM-proposed definition:

$$n^* \;=\; 1/\boldsymbol{N_\mathrm{A}} \;\approx\; 1.660\,538\,921 \times 10^{-24}\ \mathrm{mol}.$$

- Comments:

It would be better if the atomic-scale unit, entity (symbol **ent**), were to be taken as the appropriate invariant reference quantity for amount of substance (with dimension: N) rather than the Avogadro constant, $N_{\mathbf{A}}$ (= 1 $\mathbf{ent}^{-1}$), as the invariant for reciprocal amount of substance (with dimension: N^{-1})—a difficult concept to comprehend. This way, the mole could be defined directly as an exact number of entities: mol = N^* **ent**, exactly, where N^* is a fixed constant (= 6.022 141 29 × 10^{23}, exactly).

Although not mentioned explicitly, it is taken for granted in the New SI proposals and the CIPM-proposed redefinitions that the dalton will remain defined in terms of the carbon-12 atomic mass. As explained elsewhere [11, 12], the basic mole concept imposes a strict compatibility condition on the definitions of the kilogram, mole *and* dalton, taken together, so that mol/[(0.001 kg)/Da] = **ent**, where the right-hand side is the fundamental *invariant* for amount of substance: the amount of substance of a single entity. If the mole-concept compatibility condition were to be satisfied, with the redefined (fixed-h) kilogram and redefined (exact-N^*) mole, the compatible dalton *should* be given by:

$$\mathrm{Da} \;=\; (1/N^*)\ \mathrm{g} \;=\; 1/(1000N^*)\ \mathrm{kg,\ exactly,}$$

i.e., using the current best estimate of N^*, this would be:

$$\mathrm{Da} \;=\; 1/(6.022\ 141\ 29 \times 10^{26})\ \mathrm{kg,\ exactly,}$$

which, of course, is in direct conflict with the carbon-12-based definition of the dalton.

2.7 Unit of luminous intensity: candela

- Current definition:

The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 × 10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

- Current definition in explicit-*constant* format:

$$\boldsymbol{K}_{\mathbf{cd}} \equiv 683 \text{ lm W}^{-1} = 683 \text{ W}^{-1} \text{ cd sr.}$$

- Current definition in explicit-*unit* format:

The candela, cd, is the unit of luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} Hz, equal to exactly $[1/(683 \times 299\,792\,458^2 \times 9\,192\,631\,770)][\boldsymbol{m(\mathcal{K})}\, \boldsymbol{c_0}^{\mathbf{2}} \boldsymbol{\nu}_{\mathbf{Cs}}\, \boldsymbol{K}_{\mathbf{cd}} \text{ sr}^{-\mathbf{1}}]$; i.e.,

$$\text{cd} \equiv [1/(683 \times 299\,792\,458^2 \times 9\,192\,631\,770)][\boldsymbol{m(\mathcal{K})}\, \boldsymbol{c_0}^{\mathbf{2}} \boldsymbol{\nu}_{\mathbf{Cs}}\, \boldsymbol{K}_{\mathbf{cd}} \text{ sr}^{-\mathbf{1}}]$$

$$\approx 1.772\,139\,993 \times 10^{-30} [\boldsymbol{m(\mathcal{K})}\, \boldsymbol{c_0}^{\mathbf{2}} \boldsymbol{\nu}_{\mathbf{Cs}}\, \boldsymbol{K}_{\mathbf{cd}} \text{ sr}^{-\mathbf{1}}].$$

- Characteristic luminous intensity for the current definition:

$$I_{\text{v}}^* = \boldsymbol{m(\mathcal{K})}\, \boldsymbol{c_0}^{\mathbf{2}} \boldsymbol{\nu}_{\mathbf{Cs}}\, \boldsymbol{K}_{\mathbf{cd}} \text{ sr}^{-\mathbf{1}} \approx 5.642\,895\,053 \times 10^{29} \text{ cd.}$$

- CIPM-proposed (explicit-*constant*) definition:

The candela, cd, is the unit of luminous intensity in a given direction; its magnitude is set by fixing the numerical value of the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz to be equal to exactly 683 when it is expressed in the units $\text{s}^3 \text{ m}^{-2} \text{ kg}^{-1}$ cd sr, or cd sr W^{-1}, which is equal to lm W^{-1}; i.e.,

$$\boldsymbol{K}_{\mathbf{cd}} \equiv 683 \text{ lm W}^{-1}.$$

- Equivalent definition in explicit-*unit* format:

The candela, cd, is the unit of luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} Hz, equal to exactly $[1/(683 \times 6.626\,069\,57 \times 10^{-34} \times 9\,192\,631\,770^2)]\, [\boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}{}^{\mathbf{2}}\, \boldsymbol{K}_{\mathbf{cd}} \text{ sr}^{-\mathbf{1}}]$; i.e.,

$$\text{cd} \equiv [1/(683 \times 6.626\,069\,57 \times 10^{-34} \times 9\,192\,631\,770^2)]\, [\boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}{}^{\mathbf{2}}\, \boldsymbol{K}_{\mathbf{cd}} \text{ sr}^{-\mathbf{1}}]$$

$$\approx 2.614\,830\,711 \times 10^{10} [\boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}{}^{\mathbf{2}}\, \boldsymbol{K}_{\mathbf{cd}} \text{ sr}^{-\mathbf{1}}].$$

- Characteristic luminous intensity for CIPM-proposed definition:

$$I_v^* = [\boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}^{\mathbf{2}}\boldsymbol{K}_{\mathbf{cd}}\,\mathrm{sr}^{\mathbf{-1}}] \approx 3.824\,339\,357 \times 10^{-11}\ \mathrm{cd}.$$

- Comments:

The CIPM-proposed definition is formally equivalent to (a reworded form of) the current definition but, since both definitions couple the second, metre and kilogram, the characteristic luminous intensities are significantly different (approximately 5.6×10^{29} cd compared with 3.8×10^{-11} cd) because of the *redefined* (fixed-Planck-constant) kilogram. It is important to note the appearance of the steradian—an anomaly, since it is *not* a reference quantity—in both the current and CIPM-proposed definitions. The steradian is related to the radian by sr = rad$^{\mathbf{2}}$ [13, 14], where the radian is the unit of plane angular displacement. Strictly speaking, this would require introducing an eighth base unit, radian, using an appropriate reference quantity such as one full revolution, **rev**; e.g., rad $\equiv$ $(1/2\pi)$**rev**. In this case, the steradian would be a derived unit given by sr = rad^2 = $(1/4\pi^2)$**rev**^{2}. And the explicit-unit form of the CCU-proposed candela definition would be:

$$\mathrm{cd} \equiv [4\pi^2/(683 \times 6.626\,069\,57 \times 10^{-34} \times 9\,192\,631\,770^2)]\, I_v^*$$

$$\approx 1.032\,293\,788 \times 10^{12}\, I_v^*$$

where $I_v^* = \boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}^{2}\boldsymbol{K}_{\mathbf{cd}}\mathbf{rev}^{-2} \approx 9.687\,164\,757 \times 10^{-13}$ cd. However, in current SI terminology, the radian and steradian are both considered to be derived units given by rad = m m^{-1} and sr = m^2 m^{-2}, and can each be replaced by 1, thereby avoiding the necessity of introducing an eighth unit.

3. Summary

The following summarizes for easy reference, in compact form, the CIPM-proposed definitions of the seven SI base units in explicit-constant and explicit-unit formats, with the respective characteristic quantity, as well, in each case, using the currently best known numerical values of the respective reference constants.

(i) second

$$\boldsymbol{\nu}_{\mathbf{Cs}} \equiv 9\,192\,631\,770 \text{ Hz},$$

$$\mathrm{s} \equiv 9\,192\,631\,770\, t^*,$$

$$t^* = 1/\boldsymbol{\nu}_{\mathbf{Cs}} \approx 1.087\,827\,757 \times 10^{-10} \text{ s}.$$

(ii) metre

$$\boldsymbol{c}_{\mathbf{0}} \equiv 299\,792\,458 \text{ m s}^{-1},$$

$$\mathrm{m} \equiv (9\,192\,631\,770/299\,792\,458)\, l^*$$

$$\approx 30.663\,318\,99\, l^*,$$

$$l^* = \boldsymbol{c}_{\mathbf{0}}/\boldsymbol{\nu}_{\mathbf{Cs}} \approx 3.261\,225\,572 \times 10^{-2} \text{ m}.$$

(iii) kilogram

$$\boldsymbol{h} \equiv 6.626\,069\,57 \times 10^{-34} \text{ J s},$$

$$\mathrm{kg} \equiv [299\,792\,458^2/(6.626\,069\,57 \times 10^{-34} \times 9\,192\,631\,770)]\, m^*$$

$$\approx 1.475\,521\,529 \times 10^{40}\, m^*,$$

$$m^* = \boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}/\boldsymbol{c}_{\mathbf{0}}^{\mathbf{2}} \approx 6.777\,264\,719 \times 10^{-41} \text{ kg}.$$

(iv) ampere

$$\boldsymbol{e} \equiv 1.602\,176\,565 \times 10^{-19} \text{ C},$$

$$\mathrm{A} \equiv [1/(1.602\,176\,565 \times 10^{-19} \times 9\,192\,631\,770)]\, I^*$$

$$\approx 6.789\,687\,110 \times 10^{8}\, I^*,$$

$$I^* = \boldsymbol{e}\boldsymbol{\nu}_{\mathbf{Cs}} \approx 1.472\,821\,919 \times 10^{-9} \text{ A}.$$

(v) kelvin

$\boldsymbol{k} \equiv 1.380\,6488 \times 10^{-23}$ J K^{-1},

K $\equiv [1.380\,6488 \times 10^{-23}/(6.626\,069\,57 \times 10^{-34} \times 9\,192\,631\,770)]\, T^*$

$\approx 2.266\,665\,135\, T^*$,

$T^* = \boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}/\boldsymbol{k} \approx 4.411\,767\,688 \times 10^{-1}$ K.

(vi) mole

$\boldsymbol{N}_{\mathbf{A}} \equiv 6.022\,141\,29 \times 10^{23}$ mol^{-1},

mol $\equiv 6.022\,141\,29 \times 10^{23}\, n^*$,

$n^* = 1/\boldsymbol{N}_{\mathbf{A}} \approx 1.660\,538\,921 \times 10^{-24}$ mol.

(vii) candela

$\boldsymbol{K}_{\mathbf{cd}} \equiv 683$ lm W^{-1},

cd $\equiv [1/(683 \times 6.626\,069\,57 \times 10^{-34} \times 9\,192\,631\,770^2)]\, I_{\mathrm{v}}^*$

$\approx 2.614\,830\,711 \times 10^{10}\, I_{\mathrm{v}}^*$,

$I_{\mathrm{v}}^* = \boldsymbol{h}\boldsymbol{\nu}_{\mathbf{Cs}}^{\mathbf{2}}\, \boldsymbol{K}_{\mathbf{cd}}\mathrm{sr}^{\mathbf{-1}} \approx 3.824\,339\,357 \times 10^{-11}$ cd,

where sr = rad^2 = (m m^{-1})^2 = 1.

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