SOME ILLOGICALITIES OF THE SI

While the CCU and others examine the minutiae of proposals to redefine some of the base units of the SI to make them all 'granular', there is talk of a New SI; it is to incorporate in the SI those proposals. It is not to re-examine the 7th edition of the SI Brochure [1] in its entirety, to test, to correct or to reaffirm its logic. That logic is the embodiment of metrological orthodoxy and therefore assumed not to be open to question or challenge. I do not enter the various controversies surrounding the definition of the kilogram; as a process engineer I do not have the right kind of education for that. Nevertheless I append some thoughts, most of which have been published over the years by courageous editors despite being rubbished by referees: rubbished without argument but simply on the grounds that at root they are in conflict with orthodoxy, and so 'must be wrong'.

Dimension

The term *dimension* is much in evidence in the SI Brochure [1].

Derived quantities are generally defined by algebraic equations rather than by speech or by discursive statements; and equations require careful interpretation because of their short-hand nature. For example a force is commonly defined as $f = mlt^{-2}$: an equation derived from Newton's second law. It means that a constant force f applied to a mass m causes the distance moved in the direction of the force in time t_1 to change by the amount l in time t_2, more correctly expressed as

$$f = ml/(t_1t_2).$$

Its unit, newton, is kilogram metre per second per second.

If the italic symbols for base quantities are replaced by sans serif roman capitals, that equation appears as $\mathsf{F} = \mathsf{ML}/(\mathsf{T}_1\mathsf{T}_2)$. Merely changing the typeface of the symbol does not, of course, redefine what it is a symbol of, but the orthodox view is that in this case it does. In the new typeface the symbol becomes not for a quantity but for something called a 'dimension', and the orthodox representation of the term is

$$\dim f = \mathsf{ML}/\mathsf{T}^2.$$

That expression tells the reader nothing about the kind-of-quantity *force*. The VIM's [2] definition of **dimension** is 'dependence of a given quantity on the base quantities of a system of quantities, represented by the product of powers of factors corresponding to the base quantities.' The statement leaves in the air the nature of the 'dependence' or how, in representing it, the 'factors' correspond to base quantities raised to powers. If the 'factors' are not themselves base quantities, then what are they? Johansson [3] answers that question with the suggestion that the symbol L, for example, *is itself* the dimension of the kind-of-quantity called 'length': not a symbol of it, or of anything else. In other words (mine) it is no more than a special mark on paper. Then he goes on to say that it *expresses* 'the dependence of a given quantity etc', but I am no clearer as to how. Johansson makes great play with the concept *dimension* in ways that I fail to understand [4].

One is not a base quantity, yet, when the 'factors' resolve to 1 it is represented in the SI as the dimension of a dimensionless quantity; and the quantity is now more often called a 'quantity of dimension one'. A dimensionless quantity is thus said not to be 'dependent' on *any* base quantity. A relative length is deemed to be independent of the concept *length* as a kind-of-quantity. In fact all the quantities of the International System of Quantities [5], and for which the SI provides units, are ratio quantities, and therefore relative quantities in the sense that a value is represented as the ratio of one quantity to a second quantity of the same kind: a named unit.

'Dimensionless' quantities and the unit 'one'

The commonest kind of 'dimensionless quantity' is a ratio of two quantities of the same kind.

According to the SI 'The coherent SI unit of all such dimensionless quantities…is the number one, since the unit must be the ratio of two identical SI units.'   'Must be…' we are told in the SI Brochure [1]. But by what logic can a metre, say, be divided by itself or by another metre? How, physically or in reality, can it be done? The ratio of the length of the circumference of a circle to that of its diameter is the number represented by the symbol π. It is a pure number equal to 3.1415926.. : not that number of ones. Units have no relevance and are not needed to express a value of π. The SI Brochure declares the 'number one [to be] a dimensionless derived unit'. That has become today's orthodoxy, but it is contrary to reason [6]. Units cannot be multiplied by each other or divided by other units. There is no such thing in logic as 'unit algebra', though it is sometimes convenient to present units, for brevity, like algebraic expressions. Algebra has been defined as 'the generalization of arithmetic to variable or abstract quantities represented by symbols' [7]. Units are neither variables nor abstract. For example a unit of speed is metre per second, conventionally written as m/s or m s^{-1}. In such an expression the solidus (/) always means 'per', as does the negative index ($^{-1}$). 'Per' is not synonymous with 'divided by'. References [3], [6], [8] and [9] deal more widely with dimensionless quantities that are not simple ratios of two quantities of the same kind, to which the same principles apply. The illogicalities of 'unit algebra' and the 'unit one' should not be features of the New SI.

Plane angle and solid angle

In the late 1960s the sages of ISO/TC12 and the CCU were undecided about the definitions of plane and solid angles; they had designated their units as "supplementary"—neither base nor derived. In 1969 the CIPM considered that the resolution designating them thus 'allowed the freedom of treating the radian and steradian as SI base units.' In 1980 it considered that '*in the equations used <u>one</u> generally expresses plane angle as the ratio of two lengths and solid angle as the ratio between an area and the square of a length, and consequently these quantities are treated as dimensionless quantities.*' [1] That 'one' is in the Brochure's translation of the official

French text, which uses ‘*on*’. Although *on* is grammatically impersonal it is invariably interpreted by French speakers as ‘we’, meaning here ‘we scientists’.

The SI Brochure does not define plane angle; it says merely that it is the ratio of two lengths [1]. In the ISQ [5] it is defined as the ratio of the length of an arc of a circle to that of the circle’s radius, with no mention of the location of the point at which that angle is defined. In a discussion paper submitted some years ago to the CCU the “plane angle ϑ [is defined] by the equation $\vartheta = s/r$ where *s* is the arc distance around a circle of radius *r which subtends an angle* ϑ at the centre of the circle.” The definition is circular; it defines plane angle by reference to itself.

The CIPM’s recommendation was adopted by the CGPM in 1995. That resolution saddled the SI with a unit for plane angle: one, also to be called ‘radian’, which is not a unit of measurement [10]. No one measures angle in radians; it is impractical. The entire world uses the unit bequeathed to it by the Babylonians some millennia ago: the degree; or it uses the turn for angles greater than 360 degrees. The radian is used by physicists and engineers for cyclic phenomena; and the numerical part of a value of an angle of rotation is expressed as a multiple or sub multiple of π, without stating the unit. Indeed the presence of π in the expression of a value is normally sufficient to indicate that the period of the cycle, if the cycle is a rotation, is assigned the value 2π rad. If the cyclic phenomenon is not rotational, angle has no relevance. Outside the confines of the CIPM and the milieu from which its members are drawn, plane angle is everywhere regarded as and used as a base quantity in any system of quantities [10]. Solid angle is not a base quantity, it is a two dimensional angle and its unit is derived from plane angle [11].

Entities and events

Entities, once defined, can be counted. For example an atom may be defined as any particle that is represented in the table of elements, or in a particular place in that table. Once defined, then, as such, atoms may be counted. Being subject to counting does not turn an atom into a quantity as the term is defined by metrologists [2]: *property of a phenomenon, body or substance where the property has a magnitude that can be expressed as a number and a reference*[1]*;*

An atom or a molecule is a ‘phenomenon, body or substance’, not a quantifiable property of one. Countability is not a quantifiable attribute; nor is a name. One atom is no more countable than another. It may, however, be of greater mass; or it may carry a charge; those are properties that are quantities and whose values are expressed respectively in units of mass and of unit charge, which may be positive or negative: not simply as numbers of atoms. Yet the SI Brochure [] asserts that a number of molecules is a quantity, with one as its unit. In the same way it treats an event of any specified kind, such as the completion of a cycle, as a quantity with one as its unit. The SI unit of frequency is a 1 per second, written 1/s or s^{-1}. A unit of angular velocity is radian per second (also a ‘per second’ in SI parlance); the unit for a cyclic frequency is 2π radian per

[1] It might be concluded that this definition excludes dimensionless quantities, but that was not the intention. The VIM and the SI assign the number 1 as a unit (‘reference’) to dimensionless quantities, even though that reference is rarely stated. That decision is discussed by Emerson [11] and by Johansson [3].

second (or 2π per second for the SI). Frequency is not a quantity, and certainly does not merit two different SI units

Moles and 'amount of substance'

A mole of a particular substance is defined as the number of elementary particles of that substance equal to the number of atoms in 12 gram of carbon 12. Analytical chemists have found that a useful definition because knowledge of the relative mass of the particle enables the mass of a mole of it to be calculated, and they can thus prepare 'molar' solutions without counting elementary particles. Relative masses of atoms have been determined with very acceptable uncertainties. The current best estimate of the number of atoms in 12 gram of carbon 12 is $6.022141790 \times 10^{23}$. It is now proposed that that number-of-entities, made exact, should be adopted as the new definition of mole, making it independent of units of mass. As argued above, a number of entities is not a quantifiable attribute of anything; it is not a quantity according to the international definition. Thus an 'amount of substance', which is merely a number of moles, is not a quantity, and certainly not a base quantity.

Thermodynamic temperature, entropy and heat capacity

The CIPM has once more the opportunity to put to rest the unsustainable concept of a unit of thermodynamic temperature, and the notion that on the Kelvin scale such a 'unit' is equal to a unit on the Celsius scale.

A ratio of two thermodynamic temperatures T_1/T_2 is a ratio of two energies, the input (Q_1) and rejected (Q_2) quantities of heat of an ideal, reversible Carnot cycle operating between those temperatures or, on a microscopic scale, the energies of two molecules. A particular temperature T_2 may be chosen as a fixed point in the construction of a scale of thermodynamic temperatures.

The SI defines such a scale, the Kelvin scale, wherein the temperature at the triple point of water, the scale's fixed point or defining temperature, is assigned the number 273.16. The number assigned to any other temperature on the scale is to 273.16 as the energy of a molecule at that temperature is to the energy of the same molecule at the temperature of the triple point of water. It follows that the temperature of a molecule having zero energy is at the zero of that, and any other, scale of absolute temperature. There are no units of thermodynamic temperature; every such temperature is an absolute temperature. The difference between two thermodynamic temperatures is not itself a thermodynamic temperature; nor is their sum. 2 K is a temperature that is 2 on the Kelvin scale; it is not 1 K + 1 K.

Why was such a funny number chosen for the triple point of water? The Kelvin scale was deliberately defined so that a 'degree' on the scale (the difference between the temperatures of adjacent integers on the scale) would equal one degree celsius. It need not have been so and there might have been less confusion about how the scales differ fundamentally if a number had been chosen that bore no relationship to the Celsius degree. K is not a symbol for a unit of thermodynamic temperature; it identifies a *scale* of thermodynamic temperature. But we are where we are, and a difference between two temperatures (even when neither is that of the ice point) could be called a number of 'degrees celsius'. A human body temperature, say 36.8 on the Celsius scale, should be written as 36.8 celsius, or just 36.8 C. A temperature interval ΔT should be written as ΔT deg C: never ΔT K

It has been argued that a particular thermodynamic temperature is the amount by which it differs from the scale's datum temperature, which is zero, so that all thermodynamic temperatures are in fact differences of temperature and entitled to be evaluated using a unit of temperature difference. But no definition of thermodynamic temperature *defines* it in relation to a zero datum; its lowest possible value is zero simply because the Second Law rules that its values may not be negative. The matter is of significance in relation to the definition of a unit of entropy. The quantity is defined by the equation $\mathrm{d}S = \mathrm{d}Q/T$, where S is the entropy of a system and Q is an energy transferred reversibly to it at constant thermodynamic temperature T. Because the energy $\mathrm{d}Q$ is transferred isothermally we cannot substitute in that equation $\mathrm{d}Q = C\mathrm{d}T$, where C is a heat capacity. A unit of entropy is not then a unit of energy per 'unit' of thermodynamic temperature; in thermodynamic tables an entropy is always stated in relation to an arbitrary datum temperature. Nor is it a unit of energy per unit of temperature difference, as is the case with heat capacity, where the difference of temperature is that resulting from a transfer of energy at constant volume or pressure. The SI is mistaken in equating the unit of heat capacity to a unit of entropy. Units of completely different quantities cannot be the same unit.

Thermodynamic temperature is an intensive quantity like density; but density is also a derived quantity and so it has a unit that is of a derived quantity, derived from quantities that are both extensive. Nevertheless it is an absolute quantity with an absolute zero. Densities, like absolute temperatures, cannot be meaningfully added or subtracted.

Luminous intensity

The concept of luminous intensity is of the intensity of light *perceived* by a human eye, as opposed to that actually *received* by the eye. It is the intensity of light *visible* to that eye. That is not, however, a concept that is consistent with the SI unit, the candela. Every human eye is different, so the SI abandons the physiological definition and defines the unit, candela, as a fixed fraction of the radiant intensity received from a monochromatic light source of a fixed frequency and power. That leaves unanswered the question of the luminous intensity of light received from a source of a different frequency; it has to be obtained by applying an empirically determined 'standard luminosity function' (quoted in neither ISO 31 nor the SI Brochure) that averages widely differing results obtained from real human beings by variously differing means. The SI unit is of a quantity that differs from all other base quantities in that it includes arbitrarily in its definition an impersonal, empirically derived function that purports to represent a subjective quantity.

The standard luminosity function is a ratio, so luminous intensity at a particular wavelength is defined as a fixed fraction of radiant intensity at that wavelength: a derived quantity. The candela is a non-coherent unit of power divided by solid angle. Despite its undoubted usefulness and widespread use by lighting engineers it is more than questionable whether the lumen and all the units derived from the concept, including the candela, are justifiably SI units.

A parallel might be drawn between the concepts of luminous intensity and terrestrial weight. The weight of a particular mass, say a standard kilogram, is the force required to counter the force of gravity on it. That force varies according to where the mass is, because the acceleration due to gravity varies from place to place. The weight can be made a property of the mass by redefining the term ‘weight’, by adopting a conventional, fixed, standard value of the ‘acceleration due to gravity’ unrelated to place, so that the mass has the same ‘weight’ everywhere. It would be called by the CGPM its ‘standard weight’. In the same way luminous intensity, a property that is personal to the owner of a human eye and that differs from person to person, is made instead a property of the light source, independent of that eye by replacing it by an invariable mathematical function, unrelated to the human eye that perceives the light. It might at least be renamed ‘*standard* luminous intensity’. I find it hard to imagine ‘standard weight’ ever being adopted as a base quantity of the ISQ, with its own SI unit. Nor should standard luminosity.

Acoustic engineers also have a set of units that almost parallel those of lighting engineers, including units associated with perceived loudness; but loudness, another physiological quantity, is not a base quantity of the ISQ.

The stifling power of orthodoxy

In 1955 George Sarton [12] wrote, “Truth can be determined only by the judgement of experts…Everything is decided by a small group of men…The people have nothing to say but simply to accept the decisions handed out to them. Scientific activities are controlled by universities, academies and scientific societies, but such control is as far removed from popular control as it possibly could be.” He could have added that such judgements and decisions become the orthodox truths that are written into the resolutions of intergovernmental scientific committees. Their logic is widely accepted as unquestionable.

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