# Two rogue units of the SI: the kelvin and the candela
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Quantities and the expressions of their magnitudes and their units, if any, are effectively defined by three documents: the *Vocabulaire Interntaionale de Métrologie 3$^{rd}$ Edition* (VIM3) the *SI Brochure* 8$^{th}$ Edition (SI8) and the *International System of Quantities* (ISQ). Until the publication of VIM3 every kind of quantity was required to have an associated unit, now defined as 'a real scalar quantity, defined and adopted by convention, with which any quantity of the same kind can be compared to express the ratio of the two quantities as a number (VIM3)'. The current ISQ names five kinds of quantity as 'base quantities' which are considered not to require definition because of their familiarity, and all other quantities requiring units are defined as algebraic functions of base quantities as determined by observed physical laws. This long preamble is needed because I intend to show that the SI's 'unit of thermodynamic temperature', called kelvin, is not a unit of a base quantity, and therefore not a base unit, with important consequences for physics and engineering; and indeed what the SI calls thermodynamic temperature is not a base kind of quantity.

Thermodynamic temperature is an intensive quantity and, like all such quantities, it has an absolute zero, with no negative values; and it is not additive. For example a density cannot be added to another density to produce a new density that is their sum. That is not to say that a density cannot be assigned a unit, for it has in its derivation two extensive quantities; it is a mass per unit of volume. Thermodynamic temperature is called a SI base quantity; it is not 'per' any other quantity. In classical, equilibrium thermodynamics it is defined by reference to an ideal heat-engine such as that operating on a Carnot cycle. A quantity of heat $Q_1$ is input isothermally to the engine at temperature $T_1$ and a quantity $Q_2$ is withdrawn isothermally at temperature $T_2$ The amount of work done by the cycle is $Q_1$ - $Q_2$, and $T_1/T_2 = Q_1/Q_2$ . The work done is a maximum when $Q_2 = 0$ and when therefore $T_2$ also equals zero: its lowest possible value.

If a Carnot cycle operating between temperatures $T_1$ and $T_2$ receives heat $Q_1$ from another Carnot cycle operating between temperatures $T_3$ and $T_1$, the upper temperature $T_3$ cannot be said to be a thermodynamic temperature. Its lowest possible temperature is not zero; it is $T_1$. It is not possible to increase a thermodynamic temperature by addition

$T_1 + \delta T$ is not a thermodynamic temperature. An increment of temperature $\delta T$ cannot be adopted as a unit of thermodynamic temperature; every thermodynamic temperature has its foot on absolute zero as its datum point. Its foot cannot be detached from zero. If it is, the temperature ceases to be thermodynamic. Thermodynamic temperatures cannot be piled one on top of another and be

called units of thermodynamic temperature. A temperature of 2 K is not 1 K + 1 K; it is 2 on the Kelvin scale of temperature. The SI is wrong in assigning a unit (kelvin) to the Kelvin scale, which has no unit. That unit is of temperature difference.

The SI assigns the same unit to two different scalar quantities: heat capacity and entropy. By definition of the term unit the units of the quantities cannot be identical or the two quantities cannot differ. Heat capacity is defined as

$$C = \mathrm{d}Q/\mathrm{d}T$$

at constant pressure, where $\mathrm{d}T$ is an increment of temperature: a difference of temperature. Entropy is defined by the equation:

$$\mathrm{d}S = \mathrm{d}Q/T$$

where $T$ is the thermodynamic temperature at which $\mathrm{d}Q$ is added. $T$ is not a temperature difference and has no unit; it is very different from the quantity called temperature difference.

In the proposed redefinition of the kelvin for the New SI no distinction is made between thermodynamic temperature and interval of temperature. Temperatures on the Kelvin Scale and the Celsius Scale are both called 'thermodynamic'. If that were acceptable the adjective would serve no purpose and might as well be dropped. What the New SI defines as the unit of thermodynamic temperature is in fact a unit of temperature difference: the difference between temperatures separated by one on the Kelvin Scale or the Celsius Scale. If it were possible to realise temperatures on the Kelvin Scale the symbol K might be used, not to identify a unit but the scale itself.

None of this has much relevance to the *measurement* of temperature. The current definition of the Kelvin scale has one realisable fixed point: the triple point of water of a specified isotopic composition, and one that is not realisable: zero. All other temperatures remain undefined. Or rather they are defined theoretically by the properties of non-existent ideal gases to interpolate between the fixed points, and practically by reference to the International Temperature Scale ITS-90. The latter has eighteen fixed points and complicated mathematical equations to interpolate between them. ITS–90 is not a thermodynamic scale at all; it is an *equipment calibration standard*. However, all measurements of temperature are made with reference to that standard and are generally expressed in 'degrees kelvin' with no reference to the SI definitions. A value of a temperature is stated as a number and a reference: ITS-90, in the manner of hardness or octane number. The measurement and evaluation of cryogenic temperatures extremely close to absolute zero is a specialised science outside the scope of the SI.

In the proposed New SI the triple point of water is abandoned as the fixed point on the Kelvin Scale in favour of a fixed (exact) value of the Boltzmann constant, expressed as $1.380\,65 \times 10^{-23}$ joule per kelvin ($JK^{-1}$). Here again we find the SI using kelvin as though it

were a unit of absolute temperature, and the same objections must apply.

Thermodynamic temperature should cease to be a base quantity of the SI and the kelvin should not be recognised as a unit .

The concept of luminous intensity is of the intensity of light *perceived* by a human eye, as opposed to that actually *received* by the eye.  It is the intensity of light *visible* to that eye.  That is not, however, a concept that is consistent with the SI unit, the candela.  Every human eye is different, so the SI abandons the physiological definition and defines the unit, candela, as a fixed fraction of the radiant intensity received from a monochromatic light source of a fixed frequency and power.  That leaves unanswered the question of the luminous intensity of light received from a source of a different frequency; it has to be obtained by applying an empirically determined 'standard luminosity function' (quoted in neither ISO 31 nor the SI Brochure) that averages widely differing results obtained from real human beings by variously differing means.  The SI unit is of a quantity that differs from all other base quantities in that it includes arbitrarily in its definition an impersonal, empirically derived function that purports to represent a subjective quantity.

The standard luminosity function is a ratio, so luminous intensity at a particular wavelength is defined as a fixed fraction of radiant intensity at that wavelength: a derived quantity.  The candela is a non-coherent unit of power divided by solid angle.  Despite their widespread use by lighting engineers the lumen and all the units derived from the concept, including the candela, should not be described as base quantities and units of the SI.

A parallel might be drawn between the concepts of luminous intensity and terrestrial weight.  The weight of a particular mass, say a standard kilogram, is the force required to counter the force of gravity on it.  That force varies according to where the mass is, because the acceleration due to gravity varies from place to place.  The weight can be made a property of the mass by redefining the term 'weight', by adopting a conventional, fixed, standard value of the 'acceleration due to gravity' unrelated to place, so that the mass has the same 'weight' everywhere.  It would be called its 'standard weight'.  In the same way luminous intensity, a property that is personal to the owner of a human eye and that differs from person to person, is made instead a property of the light source, independent of that eye by replacing it by an invariable, highly empiracle, mathematical function, unrelated to the human eye that perceives the light.  It might at least be renamed '*standard* luminous intensity'.   I find it hard to imagine 'standard weight' ever being adopted as a base quantity of the ISQ, with its own SI unit.  Nor should standard luminosity and the candela.

Acoustic engineers also have a set of units that almost parallel those of

lighting engineers, including units associated with perceived loudness; but loudness, another physiological quantity, is not a base quantity of the ISQ.

## References

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**April 7, 2014**