(The paper below was when written meant for a philosophy of science journal and its readers, who normally are not acquainted with metrology. My intention, however, turned out to come in conflict with the more important aim that – hopefully – the paper might influence what the ninth edition of the SI brochure will be like. Therefore, the paper is now published at *Metrology Bytes* without ever has being sent to a journal. Probably, already at the end of 2015 everything around the New SI will in practice be settled. This version was finalized June 16, 2014. It can be freely distributed. Metrologists not familiar with the philosophy of science can read parts of the paper as an introduction to some issues in the philosophy of science.)

Constancy and Circularity in the SI

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Abstract The International System of Units (SI) tries to find or construct something that does not change with time and place, since such constancy is the best possible ground for definitions of fundamental measurement units. This problem of constancy has received scant attention within the philosophy of science, but is the topic of the paper. The paper first highlights inevitable kinds of circularities, semantic and epistemic, that belongs to the search for constancy, and then discusses contingent dependencies between unit definitions. The New SI proposal is criticized for not paying due attention to the important fact that it defines units (e.g. meter) for one kind of quantity (length) by means of a constancy that belongs to another (velocity). This inattention flaws the kilogram definition. The New SI definitions of the mole and the second neglect the distinction between discrete and continuous quantities; which make the definitions refer to constancies that are not invariants of nature.

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1. The Problem of Constancy

The existence of standardized measurement units is one the most essential presuppositions for the globalization of science, technology, industry, and trade. To make all these standardizations even more computer interoperable is an imminent task (Foster 2010, 2013). The ability to correctly and effectively communicate and compare the same kind of properties both on different places and at different times is crucial to modern civilization. Crucial to the establishment of good measurement units is to find or construct something that can be regarded as being constant. What is one meter ought to be one meter everywhere and at all times. Meter sticks ought neither to expand nor contract; and if they do, there should somewhere in a calibration hierarchy be a prototype or a naturally given length standard that – presumedly – never changes length; and against which the changing sticks now and then can be re-calibrated.

If I am allowed to use the modern term *metrologist* anachronistically, then metrologists have for thousands of years been searching for constancies that can ground fundamental measurement units (Crease 2011). I will call the search *the problem of (spatiotemporal) constancy*; and I find it a bit remarkable what small attention the problem has received in the philosophy of science. The explanation, I guess, is that the problem of such constancy is very much a substantial problem, and that philosophers of science have mainly rested content with discussing formal-logical and set-theoretical features of measurement procedures.[1] This paper

[1] For a check, look for instance at the classic (Hempel 1952: ch. 12), via the otherwise extraordinarily rich book (Kyburg 1984), to recent set-theoretical papers such as (Frigerio, Giordani, Mari 2010), (Giordani, Mari 2012), and (Rossi, Crenna 2013); see also the beginning of section 7 below. A small exception is some discussions around Reichenbach's notion of *coordinative definition* (Reichenbach 1958: §4).

will try to remedy this fact.[2] It is concerned with how the problem of constancy is, and has been, handled in the International System of Units, henceforth the SI.

The SI is the world's most renowned and used system of measurement units for physics, chemistry, and allied technological disciplines.[3] Intermittently, it upgrades its decisions as put forward in the SI Brochure; the last being the 8th edition of 2006 (SI8 2006). Focusing only on the name, the SI was born in 1960, but in essence it was born in 1875. That year in Paris seventeen states signed the Meter Convention, and also created a lasting organizational apparatus of conferences and committees for promoting the interests of international standardizations of scientific measurement units; at the centre was, and is, the body named the International Bureau of Weights and Measures (BIPM), located in Sèvre close to Paris.

In 1889 two platinum-iridium prototypes, one for the meter and one for the kilogram, were by BIPM sanctioned to define what it means to be 1 meter long and having the mass of 1 kilogram, respectively. Of course, everything that is exactly as long as the standard meter is also to be declared to be one meter, and analogously for the kilogram. For a while, prototypes rather than pre-existing natural standards were looked upon as the paradigmatic kinds of units, even though the unit for temporal duration has never been connected to an artifact. After 1889 several extensions and changes have been made in the unit conventions; and the same is true of the period after 1960. The SI divides the standard units into base units and derived units; a distinction that will be discussed in Section 6.[4]

2. The New SI

Since a number of years there is under discussion an official proposal from BIPM's Consultative Committee for Units (CCU), which is meant to pave the way for the 9th edition of the SI Brochure. As things stand now at the beginning of 2014, there might be a decision at the end of 2018. A draft of the second chapter was signed in September 2010 (SI9 Draft 2010), a specification was put forward in March 2013, and in December 2013 a preliminary version of the first three chapters appeared (SI9 Draft 2013); I will mainly discuss this last draft. The proposal is called the New SI, but it did by no means reach immediate acclamation; rather, the contrary, as witnessed by the web site *Metrology Bytes* (2012). Since 2010 and onwards, there exists among metrologists a whole spectrum of views towards the New SI.

Let me start at the positive end of the spectrum, i.e., at the proposal authors. They talk about the "extraordinary advances [that since 1960] have been made in relating SI units to truly invariant quantities such as the fundamental constants of physics and the properties of atoms" (SI9 Draft 2013: 8); and they look upon their proposal as a culmination of this trend. The New SI wants to take away the last existing material prototype, the international prototype of the kilogram, and relate all the base units to constants in fundamental laws and to properties that are invariant when derived from fundamental laws (SI9 Draft 2013: 11).

From the negative end of the spectrum I will quote a chemist and metrologist who labels himself *advocatus diaboli* (Price 2010: 421, 2011: 131a). He writes: "The choice of the fundamental constants of nature as metrological anchors, as they are understood by current science, at current best accuracy values, runs a real risk of being a Zanzibar system of cosmic

[2] I am not, however, the only contemporary philosopher of science who tries to change the situation; see e.g. (Chang 2004) and (Tal 2011).

[3] It is only concerned with scalar quantities. It has been claimed that it has to take also vector quantities into account (Emerson 2014b). My views on vector quantities can be found in (Johansson 2009).

[4] The term 'standard unit' cannot be found in either the SI Brochure (SI8 2006) or the terminological clarifications in (VIM3 2012), but it is the natural term to use as a common label for base units and derived units. Both kinds of units are measurement units (VIM3 2012: def. 1.9), but so are also multiples and submultiples of them.

proportions" (ibid.: 130a). By a Zanzibar system is meant a system where, *unknown to each other*, A checks his measuring device (in the original story a clock) by means of the device belonging to B, who, conversely, checks his instrument against A's; described in e.g. (Crease 2011: intr.). If B's device starts to malfunction after he has checked it against A's, but before A calibrates his against B's, the malfunction might not be detected.

In the Zanzibar system the measurement instruments are of the same kind (clocks), but in relation the New SI the fear is that the standard units for different kinds of quantity have become so interdependent, that their spatiotemporal constancy can no longer in any real sense be tested. The indisputable point is that the links the New SI creates between base units and fundamental constants, praised by the defenders of the New SI, bring with them a number of new interdependencies between the old units. This paper is an attempt to sort out and clearly distinguish the different kinds of circularities and mutual dependencies that exist in both parts of the older SI systems and in the New SI. Let it be said at once, there are two semantic issues that I find to be beyond all doubt: (i) it has become directly misleading to talk about a distinction between two kinds of standard units, base units and derived units, respectively (Section 6), and (ii) it is equally misleading to call the Avogadro constant a fundamental constant (Section 7).

The SI of 1960 contains six base units, but since 1971 there are seven; the number of derived units is enormous. In this paper I will comment upon the new proposals for all these base units except two, the kelvin and the candela. I will not discuss the kelvin, the unit of temperature, since temperature is not, unlike the other base quantities, straightforwardly additive.[5] The candela will not be discussed because the quantity for which it is the standard unit, luminous intensity, does not belong to physics and chemistry proper.[6]

3. Inevitable Conceptual Circularities

Standard units, be they called base units or derived units, are always units for what the SI calls *quantities*, but what for the pedagogical purposes of this paper I will from now on (quotations aside) call *kinds-of-quantity*.[7] For instance, the second is the unit for the kind-of-quantity time duration, the meter is the unit for length, the kilogram is the unit for mass, and the meters-per-second is the unit for velocity. All base unit definitions take a kind-of-quantity for given (SI8 2006: 103). Therefore, the first thing to be noted in relation to possible circularities is that no concept of a kind-of-quantity can be understood before it is connected to other concepts.

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[5] Perhaps I should add that I am very puzzled about the true structure behind both the present and the proposed definition of the kelvin. At present it is grounded in the presumed constancy of the triple-point of water, and the New SI wants to ground it in the Boltzmann constant. I share the critical views put forward in (Emerson 2014a). There is a fine book on early temperature scales, (Chang 2004), but it ends before the kelvin is introduced.

[6] Unfortunately, at least from a pedagogical point of view, this fact is not explicitly stated when the definition of the candela is presented (SI8 2006: sect. 2.1.1.7); it is said only later in an appendix. There it is said that "the only photobiological quantity which has been formally defined for measurement in the SI is for the *interaction of light with the human eye* [italics added] in vision" (SI8 2006: 173), and that this quantity is luminous intensity. It has rightly been remarked that "the candela is a *physiological unit* [italics added] defined for one wavelength, and should not be a base unit in the SI" (Foster 2013: 530b). On the other hand, *if* the implicit mentioning of the human eye is abstracted away, then "The candela is clearly a derived unit" (Foster 2010: R47b). For similar remarks see also the second half of (Emerson 2014a). The New SI states in passing: "The luminous efficacy K_{cd} is a technical constant related to a conventional spectral response of the human eye" (SI9 Draft 2013: 18).

[7] The SI Brochure does not contain the term 'kind of quantity', but it refers (SI8 2006: 103) for terminological clarifications to VIM3; and VIM3 contains a definition of the term (VIM3 2012: def. 1.2). The hyphenation is not in VIM3, but it is not an invention of mine. I have taken it from the distinguished metrologist René Dybkaer (2009), who intends his book to be in conformance with VIM3.

It is always the case that in order to understand a certain concept we also have to understand a bunch of other but related concepts. This non-atomistic view of concept meaning is part and parcel of modern philosophy of language. It is even a view that crosses some traditional philosophical divisions. Within analytic philosophy, it can very explicitly be found in philosophers as different as Ludwig Wittgenstein, W.V.O. Quine, and Donald Davidson; and within non-analytic philosophy a semantic non-atomism is fundamental to both hermeneutic philosophy and structuralism. It permeates modern cognitive science, too.

Trivially, we cannot understand "to the left" without understanding "to the right," and vice versa. We have to move in a circle and learn both the concepts simultaneously. Similarly, but a bit more complex, no one can learn what "mass" means without at the same time learning about some other concepts in Newtonian mechanics. In order to understand what "mass" means we must understand at least the difference between the concepts "mass" and "weight," and in order to understand this difference, we have to learn a little also about the concept of "gravitational force." The concept of "mass" cannot be introduced by means of a definition where all the defining terms are already understood old everyday concepts.

Here is another simple example. For a long time, the basic unit for measuring temporal duration was the solar day. But the concept of solar day relies on concepts of spatial distances. A solar day is the time interval between two successive appearances of the Sun at its *highest point* in the sky. In a similar way, anyone trying to understand a concept that refers to one of the present base kinds-of-quantity of the SI will have to learn a network of concepts.

No definition of a standard unit can possibly be regarded as simultaneously supplying a definition of the kind-of-quantity that it is construed to be the standard unit for. "The unit is simply a particular example of the quantity concerned" (SI8 2006: 103). A standard unit can be defined only when there is a pre-given understanding of a concept that refers to a kind-of-quantity. And, always, this concept is part of some conceptual networks where loops are the rule rather than the exception.

I will call this fact *the semantic-holistic predicament of human measurement*. Necessarily, all editions of the SI Brochure have conformed to it, and all future editions will.

4. Inevitable Epistemic Circularities

During the twentieth century, not only the philosophy of language moved from atomistic to non-atomistic views; the same happened with the view on empirical testing in the philosophy of science. Karl Popper stressed the need for background knowledge, Thomas Kuhn coined the term paradigm, and Imre Lakatos that of research program. This development took place in parallel with a growing general acceptance of Quine's expression "the underdetermination of scientific theories by empirical evidence." This means that even the presumed constancies of *c* (the scalar velocity of light in vacuum) and *h* (the Planck constant) are underdetermined by empirical evidence. The first constant is part and parcel of the relativity theories, and the second of the quantum mechanical theories. Empirical science is fallible through and through; the presumed constancies of *c* and *h* are no exceptions. But the same is equally true of the constancy of metrological prototypes. Let me explain.

The present SI definition of the kilogram can be given this simple verbal form:

1 kilogram mass $=_{def}$ the mass of the platinum-iridium cylinder at BIPM
(and then, by implication, all other bodies that have the same or an exactly similar mass have a mass of 1 kg, too[8]).

[8] As pointed out in (Johansson 2010: 220–221), the SI can be given both a realist and a nominalist reading of its property talk; therefore, there is no need to bring in the realism-nominalism issue here.

As pointed out in the former section, in order to understand this definition we have already to understand what mass is. But there is more complexity to this seemingly straightforward definition than the conceptual circles that come with the mere concept mass. The purely verbal definition does not mention why the cylinder is made by platinum-iridium, why its edges have the form they have, why there are handling instructions, and why there are six official copies of the prototype against which the prototype can be compared. The kind of *surround knowledge* (if I may coin a term) that is used in order to secure that the prototype does not change mass over time, is not mentioned in the explicit verbal definition, but without such knowledge the definition would be completely gratuitous, and thereby useless. A reasonable prototype definition of the kilogram must implicitly rely on knowledge of other kinds-of-quantity than that of mass; in particular, on some law-like connections between these and possible changes of mass.

No construction and definition of a SI prototype can possibly avoid the kind of knowledge dependence between kinds-of-quantities now noted. All material prototypes are in principle vulnerable to deterioration or damage. Now, of course, the fundamental constants *c* and *h* are not vulnerable to such changes; the uncertainty of their constancy comes, as said, from the empirical underdetermination of scientific theories. However, not even on the assumption that *c* and *h* will forever be part of our best tested theories, is there an epistemic gulf between *constant based* unit definitions and *prototype based* unit definitions. A constant based definition would be completely useless if there are no scientists who know how to put the definition into practice. Each constant based definition must be, and is, complemented by a so-called *mise en pratique*, i.e., a set of instructions of how to create a primary realization of the definition in question (the French expression is used also in English SI texts). The knowledge contained in such instructions can be called a kind of surround knowledge, too.

In the present SI, the *mise en pratiques* are presented in an appendix that is available only online (SI8 2006: 172), but I guess they will be given a more central position if the New SI (or something like it) becomes accepted. My reason is that the proposal says that it "effectively decouples the definition and practical realization of the units" (SI9 Draft 2013: 9), and so I think it cannot rest content with talking only about definitions.[9]

Put briefly, *what the handling instructions are to a prototype based standard unit, the primary* mise en pratiques *are to a constant based standard unit*. Measurement units cannot possibly be wholly anchored in a theoretical realm; not even if the empirical underdetermination of theories is neglected. Surround knowledge of the *mise en pratique*-kind will still be needed.

What I have now described might be called *the double epistemic-fallibilistic predicament of human measurement*. (The term double indicates, on the one hand, a reference to the fallibility of the testing of the theories behind constant based unit definitions and the construction knowledge behind the prototypes, and, on the other hand, a reference to the fallibility of the *mise en pratiques* of constant based units and the handling instructions of the prototypes.) Necessarily, all editions of the SI brochure have conformed to this predicament, and all future editions will.[10]

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[9] In the philosophy of science, the necessity of such realizations is stressed in (Tal 2011).

[10] My notion "epistemic circularity" must by no means be conflated with the notion "epistemic iteration" put forward in (Chang 2004). In an epistemic iteration, one epistemic circularity is exchanged for another and presumably better one. Chang's views conform well to the claims I make in the concluding section, "The Possibility of Metrological Improvements."

5. Contingent Dependencies between Base Units

The inevitable semantic and epistemic dependencies highlighted in the preceding sections should be kept distinct from the kind of *contingent* dependence I will mention next, namely the kind of dependence that exists when a base unit in its definition explicitly or implicitly mentions some other base unit(s). Definitions of *derived* units must of course in some way mention the base units from which they are derived, but this is the topic of the next section.

The kind of dependence now at issue has a long history. For instance, when in 1795 in France the gram was officially defined, it was decreed to be the weight of a volume of one cubic *centimeter* of pure water (at the temperature of melting ice).[11] In the present SI (leaving, as said, the kelvin and the candela aside) it looks as follows: the meter definition is dependent on that of the second; the ampere definition is dependent on those of the second, the meter, and the kilogram; and the mole definition is dependent on the kilogram. This is explicitly stated (SI8 2006: 111). In the New SI, *leaving the dependencies on fundamental constants aside*, it looks as follows: the meter definition is dependent on that of the second; the ampere definition is dependent on that of the second, the kilogram is dependent on the second and the meter, but the mole on nothing. That is, even the New SI contains some base-to-base unit dependencies. It is easily seen that the second is in some sense more basic than all the others. I will discuss the second in Section 8, but until then simply take the existence of a good definition of the second for granted.

This kind of base-to-base unit dependency is not inevitable. In principle, it would be possible to make one prototype for each base kind-of-quantity, but the existence of the inevitable epistemic circularities mentioned makes it a matter of pragmatics whether base-to-base unit dependencies should be allowed or not. I can at the moment see no general reason to try to get rid of them only because they are not inevitable. They have to be discussed on a case by case reasoning, and the same is true of the dependencies discussed in the next section.

6. Base Units as Dependent on a Constancy in a Derived Kind-of-Quantity

When prototypes are used in metrology, it becomes natural and practical to let the assumedly unchanging prototype be a direct exemplification of the standard unit, and so ascribed the number 1; as was/is the case with the meter and kilogram prototypes, respectively. But there is no theoretical necessity behind this choice. When a unit grounding constancy is found in nature, it is often natural and practical to make the standard unit a *fraction* or a *multiple* of the assumed constancy. For instance, the French Academy of Sciences declared in 1791 the meter to be the *fraction* one tenth-millionth of the assumedly constant length of the meridian going from the North Pole through Paris to the Equator; and between 1960 and 1983 the SI defined 1 meter to be equal to the *multiple* 1 650 763.73 wavelengths of an assumedly constant radiation (that corresponding to the transition in a vacuum between the $2p^{10}$ and $5d^5$ quantum levels of the krypton-86 atom). Base units are intended to be grounded in a magnitude that is spatiotemporally constant, but such a constancy needs not be ascribed the number 1 and regarded as a direct exemplification of the unit it grounds.[12]

What has just been highlighted is seldom explicitly stated in metrological writings, but it is important to keep it in mind in what follows. Below, in three different cases with one

[11] In 1799 it was exchanged for a prototype, which, in turn, 1889 was replaced by the present international prototype of the kilogram.

[12] The existence of this possibility is in conformance with the following statement: "[A] unit is simply a particular example of the quantity concerned" (SI8 2006: 103). Fractions and multiples can be examples, too.

subsection for each, dependencies between definitions of base units and derived units in the SI will be discussed.

A base unit definition is neither a definition that relates a concept to other concepts as in a *lexical definition*, nor a so-called *Aristotelian real definition*; the latter lays claim to define what the true nature of a certain kind of objects is like. A base unit definition is a *coordinative definition* (Reichenbach 1958: §4); it stipulates a relation between a concept on the one hand and a magnitude, object or kind of objects in the language-external world on the other.

Since the concepts used (e.g. 1 kg) always contain a number reference, the definitions connect numbers with the world, too. All the base unit definitions of the New SI start with a sentence that conforms to the following abstract form: "the base unit *U*, symbol u, is the SI unit of kind-of-quantity *Q*; its magnitude is set by …" By talking in terms of "the meter," "the kilogram," etc. ("the base unit *U*"), the fact that number 1 is part of the definition becomes somewhat hidden; but, surely, the number is there. Moreover, as will become clear as we move along, it is there as a number on the real number line, not just as a natural number among the other natural numbers.

6a. The Case of the Meter and the Meters-per-second

After 1889 the meter has been given three different definitions. Until 1960 it was defined as being equal to the length magnitude of the standard meter in Paris, and between 1960 and 1983 it was defined as being equal to the length magnitude of 1 650 763.73 wavelengths of a certain radiation. I have no more to say about these two definitions than what has been said in the sections about conceptual and epistemic circularities. However, since 1983 the meter is grounded in the assumption that the velocity/speed of light in vacuum is always and everywhere the same, and it is defined as follows:[13]

> The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.[14]

[And then, in a new paragraph directly after the definition, it is stated:]

> It *follows* [italics added] that the speed of light in vacuum is exactly 299 792 458 metres per second. (SI8 2006: 112)

From a logical-structural point of view, the introduction of this meter definition represents a break with older metrology, and it is this structure that is made pervasive in the New SI. Superficially, it may look as if this meter definition depends *only* on the definition of the second, but this is wrong, which reflections on the last part of the quotation can make clear. Nothing numerical about the velocity of light can possibly *follow* from the meter definition if, beside time (*t*) and length (*l*), not also velocity (*v*) is ascribed a standard unit. Surely, a simple constancy claim such as the statement "the velocity of light is always and everywhere the same" needs no unit, but that is quite another thing. Obviously, in the meter definition a specific standard unit of velocity is tacitly presupposed, namely meters-per-second.[15] In order to see the whole metrological structure of the meter definition above, one must become clear

[13] I disregard the terminology according to which "velocity" is used only for vectors and "speed" only for scalars; I treat them as synonyms. Furthermore, in the text I am using the term "meter," but in quotations I will not change "metre" to "meter"; all official SI texts use the spelling "metre."

[14] If the definition of the second is inscribed, then the meter definition becomes: The meter is the length of the path travelled by light in vacuum during a time interval of 30.6633189884984 caesium-133 hyperfine transition cycles.

[15] The hyphenation is added in order to make all unit concepts one-word concepts.

about from where the meters-per-second comes, and why its sudden appearance looks so innocent.

It is all too easy wrongly to think that since $v = l/t$, then as soon as the meter is made the unit for length and the second for time, the unit for velocity cannot be but meters-per-second. The complete structure, however, looks as follows. The formula $v = l/t$ states a numerical relation, but it is not a purely mathematical relation. The variables are variables for physical magnitudes, and the formula cannot be applied to the world before the variables are connected to measurement units. Nonetheless the formula does not mention any specific measurement units; and this is not a feature peculiar to this kinematic formula. The same goes for all the natural laws of mathematical physics; Newton's second law, $F = m\ a$, does not mention any measurement units either. We stumble upon a general problem that can be formulated thus: *how can there be numerical physical relationships without measurement units?* Let us take a closer look at $v = l/t$.

Implicitly, all the three variables are variables for ratios or proportions. The variable v represents the ratio between velocity magnitudes and *some* standard unit magnitude for velocity. Similarly, l represents the ratio between length magnitudes and *some* standard unit magnitude for length, and t represents the ratio between temporal durations and *some* standard unit magnitude for temporal duration. Or, to quote the SI: "[a] number [of a variable] is the ratio of the value of the quantity to the unit. For a particular quantity, many different units may be used" (SI8 2006: 103) (SI9 Draft 2013: 1).

Sometimes, kinds-of-quantity variables such as l, t, and v are in metrology subsumed under a generic variable called Q, and what has just been said is stated by means of the following *non-arithmetic and non-algebraic formula*: $Q = \{Q\}[Q]$.[16] Here, $[Q]$ symbolizes a unit for the kind-of-quantity Q, and $\{Q\}$ symbolizes the real numbers that correspond to the ratios between specific magnitudes and the unit $[Q]$. In other words still, to claim that a certain magnitude equals the quantity Q_1 (e.g., l_1, t_1, v_1) is to claim that it is Q_1 (e.g., l_1, t_1, v_1) *times* a given specific unit magnitude.[17]

In this symbolism, the relation $v = l/t$ becomes $\{v\}[v] = \{l\}[l] / \{t\}[t]$. And by means of this formula a fact of importance can easily be made visible. In order for there to be equality between the left and the right hand sides, we cannot put in any units we want. If we let $[l]$ be the meter, $[t]$ be the second, and $[v]$ be the yards-per-second, then $v = l/t$ is false. True is instead now: $v = 0.9144\ l/t$ (1 yard equals 0.9144 meter).

A *completely unit independent* symbolization of the relation between (mean) velocity, distance traversed, and the time of the movement cannot possibly be written $v = l/t$. However, two different kinds of changes can make the formula unit independent. Either the equality sign is exchanged for a proportionality sign: $v \propto l/t$. Or a symbol for a purely metrological *unit adjuster* (such as 0.9144 in the example) is inserted in the formula. Such a number has the function retrospectively to secure that the equality holds whatever units are chosen for the variables. If we symbolize such a retrospective unit adjuster α, then we can in general truly claim:

$v = \alpha\ l/t$ (which is shorthand for: $\{v\}[v] = \alpha\ \{l\}[l] / \{t\}[t]$).

The physical relation $v = \alpha\ l/t$ can be reduced to the equality $v = l/t$ only on the presuppositions that (i) the measurements units are chosen in such a way that $\alpha = 1$, and that

[16] It must not be read as a case of multiplication! It *cannot* be turned into $\{Q\} = Q / [Q]$. As far as I can judge, this symbolism is running out of fashion, but it is retained in passing in the New SI (SI9 Draft 2013: 11).

[17] This means that we are assuming so-called *ratio scales*. Another kind of metric scale is the linear interval scale that was used by both Celsius and Fahrenheit. This scale, however, is given no place in the SI system. For a presentation of different kinds of scales, see (Dybkaer 2009: ch. 17).

(ii) the number 1 is allowed to be absent from the formula. This kind of reduction has of course been both expedient and very fruitful in theoretical physics, but it may nonetheless invite bad metrological thinking.[18]

According to my discussion experiences, it needs now and then to be stressed that more than pure mathematics is needed in order to go from the unavoidable view that mean velocity is directly proportional to distance traversed (given the time) and inversely proportional to the time needed (given the distance). This proportionality insight is not a matter of arithmetic or geometry. That the relation $v = \alpha\ l/t$ holds is an insight belonging to kinematics, and kinematics belongs to physics; that is, the relation belongs to physics, be it trivial or not.[19]

We can now see that the meter definition apart from a time unit definition also presupposes (i) that the kinematic relation stated by $v = \alpha\ l/t$ is true, and (ii) that the unit for velocity is chosen in such a way that $\alpha = 1$. That is, the purportedly derived unit meters-per-second is already presupposed when the definition of the meter is presented. From a logical point of view, what magnitudes are to be regarded as coordinated with the concepts "1 meter" and "1 meters-per-second" become stipulated in one single coordinative definition.

That the meter and the meters-per-second magnitudes are simultaneously defined is also shown by the fact that one could equally well, without any change of substantial content, make the velocity unit look like the base unit. It is the velocity unit that directly gives expression to the unit grounding constancy chosen, the velocity *c*. As the meter once was defined as a fraction of the meridian, one can define 1 unit velocity to be the fraction 1/299 792 458 of the velocity of light; and then make the meter a derived unit by defining it by means of the formula $l = v\ t$ (just as now the meters-per-second is derived from $v = l/t$).

Before 1983 all base unit definitions for a kind-of-quantity were grounded in a magnitude constancy that belonged to the same kind-of-quantity as the unit magnitude belonged to, but in the meter definition of 1983 this is not so. Here, *the standard unit definition for one kind-of-quantity (length) is grounded in a constancy of another kind-of-quantity (velocity)*. Always when this is the case, the first standard unit must in its definition have an explicit or implicit reference to a standard unit of the second kind-of-quantity. Therefore, it is since 1983 logically wrong to call the meter a base unit and the meters-per-second a derived unit.

The New SI retains all the substantial content of the 1983-definition, but gives it a new linguistic dress. And in this it becomes quite clear that the meters-per-second unit magnitude is part of the meter definition. What length magnitudes that are to be called 1 meter are "set" by using (i) the constancy of the velocity of light, (ii) the kinematic relation $l = v\ t$, and (iii) the velocity unit meters-per-second ($m\ s^{-1}$). The definition proposal looks like this:

> The metre, symbol m, is the SI unit of length; its magnitude is set by fixing the numerical value of the speed of light in vacuum to be exactly 299 792 458 when it is *expressed in the SI unit for speed* $m\ s^{-1}$ [italics added]. (SI9 Draft 2013: 13)

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[18] What has been stated is true only when one looks at an isolated natural law. If the same variables occur in more than one law, then if α is set equal to 1 in one law, it might be impossible to make it equal to 1 in all the others. For instance, as pointed out in (Simons 2013: 520–521), if we turn Newton's second law $F = \alpha\ m\ a$ into $F = m\ a$, then it is impossible to turn his gravitational law $F = \alpha\ m_1 m_2/r^2$ into $F = m_1 m_2/r^2$. It becomes the well known $F = G\ m_1 m_2/r^2$ (or $F \propto m_1 m_2/r^2$). If, on the other hand, G is set equal to 1, then $F = \alpha\ m\ a$ cannot be reduced to $F = m\ a$. Formally, it is possible to start to regard G as a variable instead of a constant, but *within Newtonian mechanics* it is only a proportionality constant and a unit adjuster. In other words, the "Big G" of Newtonian mechanics is not a constant of nature. This fact is not always made clear.

[19] Whether, after relativity theory, the relation should be confined to the inside of an inertial frame of reference, I will not discuss. Once it was self-evident that if one body moves away from you and your inertial frame with the velocity v_1, and a second body moves away from the first in its inertial frame with the velocity v_2, then the second moves away from you with the velocity $v_1 + v_2$. According to the theory of special relativity, however, the second moves away from you with the velocity $v = (v_1 + v_2)/(1 + v_1 v_2/c^2)$.

Despite this open admittance that the meter definition contains the "derived" standard unit for velocity, the New SI asserts that all their "description[s] in terms of base and derived units remains valid, although the seven defining constants provide a more fundamental definition of the SI" (SI9 Draft 2013: 2). In particular, this means that it claims that its description of the meter as a base unit and the meters-per-second as a derived unit "remains valid." To me, this conceptualization heavily blurs the normal connotations of the words "base unit" and "derived unit." In the long run, this cannot be a good thing.

Note that this complaint of mine is only semantic, not epistemic. Both when the meter magnitude is defined by a prototype or a multiple of wavelengths and the meters-per-second really is a derived unit, and when as now the meter and the meters-per-second are immediately made interdependent, it is the same relation $v = \alpha\ l/t$ that connects the units. That is, both the pre-1983 and the post-1983 SI systems take (rightly of course!) this kinematic relation as simply given. To anticipate, in the next two cases I do not find the corresponding physical relation equally unproblematic; in the last case even highly problematic.

6b. The Case of the Ampere and the Coulomb

The present definition of the unit magnitude for the kind-of-quantity electric current is this:

> The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length. (SI 2006: 113)

Explicitly, the definition of the ampere is made dependent on one base unit (the meter) and one derived unit (the newton, unit for the kind-of-quantity force). In the same way as the meters-per-second was pre-1983 derived by means of the physical relation $v = \alpha\ l/t$, the newton unit is derived by means of Newton's second law, $F = \alpha\ m\ a$ ($\alpha = 1$); and since this law only functions in tandem with the first and the third, all three are used. This derivation presupposes units for the base kind-of-quantity mass and the derived kind-of-quantity acceleration; and acceleration presupposes in turn a unit for velocity, which presupposes units for length and time duration. In this stepwise way the present SI looks upon the newton as a unit derived from the base units the meter, the kilogram, and the second. As in the former subsection, I would like to stress that such a derivation is not in any sense a purely mathematical derivation. Apart from the base units chosen, also Newton's three laws of motion and classical kinematics are presupposed.

The ampere unit magnitude cannot in a similar way be derived from kinematics, mechanics, and the connected standard units chosen. A law from classical electrodynamics is needed, and the one used is Ampère's force law: $F_m = 2k_A\ i_1 i_2/r$. Here i_1 and i_2 are currents in two straight parallel conductors, r is the distance between them, F_m is a variable for the kind-of-quantity force-per-length-unit, and $2k_A$ represents (I would say) both a constant of nature and a unit adjuster of the kind I have symbolized α. Surely, the definition of the base unit ampere has many presuppositions, but it does not presuppose a unit for any other electrodynamic kind-of-quantity than that of electric current; in particular it does not refer to a unit for electric charge. The ampere is made the base unit of electrodynamics.

The kind-of-quantity electric charge (q) is in the present SI regarded as a unit derived by means of the electrodynamic relation: $q = \alpha\ i\ t$. In the construal α is set equal to 1, the unit chosen for i is the ampere, and the one for t is the second; the unit for electric charge, the coulomb, is then defined as 1 coulomb (C) = 1 ampere (A) × 1 second (s). The definition is often shortened to 1 C = 1 A × 1 s or to C = A s.

In the light of this, let us look at the New SI. Its definition is this:

> The ampere, symbol A, is the SI unit of electric current; its magnitude is set by fixing the numerical value of the elementary charge to be exactly 1.602 176 565 $\times 10^{-19}$ when it is expressed in the SI unit for electric charge C = A s.

The reason behind the proposal is that modern physics takes the elementary charge e (positive in protons and negative in electrons) to be unchangeable and the same everywhere, i.e., e is regarded as a true spatiotemporal constancy. So far, I have no objections, but look at the definition proposal once more. In the very definition of the purported *base* unit ampere one finds the purportedly *derived* unit coulomb, C. The structure that in the former section was shown to exist between the meter and the meters-per-second via the kinematic relation $v = l/t$, does here hold between the ampere and the coulomb via the electrodynamic relation $q = i\,t$. The electric current magnitudes that are to be called 1 ampere are "set" by using (i) the constancy of the elementary charge, (ii) the electrodynamic relation $q = i\ t$, and (iii) the electric charge unit coulomb.

From a logical point of view, the ampere unit and the coulomb unit are in the New SI simultaneously chosen and defined in a way that is analogous to that between the meter unit and the meters-per-second unit (in both the present and the New SI). Therefore, I find it here equally semantically misleading to talk about a distinction between a base unit and a derived unit. There is though an epistemic difference, the relation $q = \alpha\ i\ t$ might be easier to contest than $v = \alpha\ l/t$.

That the ampere and the coulomb are simultaneously defined is also shown by the fact that one could just as well have started by defining the coulomb unit. It is the coulomb unit that directly gives expression to the unit grounding constancy chosen, the elementary charge e. There are no problems in defining 1 unit electric charge (the coulomb) as being equal to the multiple 1.602 176 565 $\times 10^{-19}$ of the elementary charge, and then make the ampere a derived unit by defining it by means of the relation $i = e/t$ and the formula 1 A = 1 C/s (just as now the coulomb is derived from the ampere by means of 1 C = 1 A × 1 s).

Both the present SI and the New SI take a number of physical relationships as simply given when they define the ampere and the coulomb, and most of these relationships are not as indisputable as the relation between velocity, length, and time is. However, it is exactly the same relationships that are presupposed in the present SI and the New SI. Therefore, there is no epistemic difference between how the two definitions handle the ampere and the coulomb.

Before turning to the next case, I would like to highlight a special feature of the present definition of the ampere; I will refer to it in the next subsection.

When unit definitions are anchored in theories instead of in obviously existing macroscopic magnitudes such as prototypes, meridians, and solar days, then a quite special metrological possibility arises. Since theories are not only about what actually exists and can exist, but also about counterfactual situations that may never exist and may not even possibly exist, the constancy magnitude that grounds a unit definition can be placed in a non-existing or impossibly existing magnitude. This is the case in the present definition of the ampere. The definition above contains the phrase "two straight parallel conductors of infinite length," but such conductors are physically impossible. As noted in (Tal 2011: 1087), the same is true of the definition of the second. It refers to a kind of atom that has the impossible temperature state of zero degree Kelvin (see footnote 24).

6c. The Case of the Kilogram and the Joules-times-seconds

First some words about what triggered the opinion that the kilogram prototype has made its duty and should go. This is in order to substantiate my intimation in Section 1, that I find it a bit remarkable how contemporary philosophy of science has neglected the problem of constancy. Let me start with a quotation:

> A pivotal event took place in 1988, when the IPK [the international prototype kilogram] was removed from its safe and compared with the six identical copies kept with it, known as *témoins* (witnesses). [---] The verification in 1988 confirmed this trend: not only the masses of the témoins but those of practically all the national copies had drifted upward with respect to that of the prototype, [---] Quinn, who became the BIPM's director in 1988, outlined the worrying implications of the apparent instability of the IPK in an article published in 1991. Because the prototype *is* the definition of the kilogram, technically the témoins are gaining mass. But the "perhaps more probable" interpretation, Quinn wrote, "is that the mass of the international prototype is falling with respect to that of its copies"; that is, the prototype itself is unstable and losing mass. (Crease 2011: 253–255)

Whether this decision should be reckoned "Fingerspitzengefühl" or merely a kind of metrological common sense, I don't know, but that is unimportant. Here is a quotation from a philosophical book on the foundations of science published 1919:

> If we have made many copies of a unit and found, just after they were made, that they and the unit were all equal, and if we find later that the copies and the unit are not still equal, then we can say either that the copies have changed or that the unit has changed. If all the copies (or nearly all) are still equal to each other, though differing from the unit, then we shall say that the unit has changed; (Campbell 1957: 362)

This kind of sensitivity to the problem of constancy is mostly absent or abstracted away in the philosophy of science after 1950. There is then only a simple noticing that a standard unit is needed and has to be chosen. And Carl Hempel is in his classic treatise using the very kilogram prototype as his example. Without any qualifications at all he simply states: "A specific object *k*, the International Prototype Kilogram, is to serve as a standard and is to be assigned the *m*-value 1,000" (Hempel 1952: 63).

The kilogram definition of the New SI looks like this:

> The kilogram, symbol kg, is the SI unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be exactly $6.626\ 069\ 57 \times 10^{-34}$ when it is expressed in the SI unit for action J s = kg m^2 s^{-1}. (SI9 Draft 20013: 13)

This means that what mass magnitudes should be called 1 kilogram are "set" by using (i) the Planck constant, (ii) some physical relationships, and (iii) the standard unit for action, joules-times-seconds (J s). The kind-of-quantity action is defined as being energy *multiplied* by time; compare velocity, which is length *divided* by time. In many contexts it is expedient to call both multiplication and division multiplications (using the rule $1/a = 10^{-a}$), but in the metrological context at hand such a wide multiplication notion easily hides things.

I have shown that the meter definition also simultaneously defines the velocity unit meters-per-second, and that the New SI's ampere (A) definition simultaneously defines the charge unit coulomb (C). In the same way the New SI's kilogram (kg) definition simultaneously defines the action unit joules-times-seconds (J s). Compare the end of the ampere definition, C = A s, with the end of the kilogram definition, J s = kg m^2 s^{-1}. In the meter case, the physical relationship that makes a two-sided coordinative definition possible is a basic purely kinematic relationship, and in the ampere case the corresponding relationship is purely electrodynamic, but what does the relationship look like in the kilogram case?

The metrologists behind the New SI have found the relationship wished for in $m = hv/c^2$. However, in the last draft it is not said from where or how they have found it; it is presented as being an epistemologically indubitable relationship (SI9 Draft 20013: 13). This I find odd, since it was said in the earlier drafts; even though two different stories were told. Let me explain and comment.

In the very first draft it is said that the two famous equations $E = mc^2$ and $E = hv$ "together lead to $m = hv/c^2$" (SI9 Draft 2010: 7). Hereby, the kind-of-quantity mass as used in relativity theory and the kind-of-quantity action from quantum mechanics become directly connected. In this operation it is taken for granted that the energy variables of relativity theory and of quantum mechanics are variables for the same kind-of-quantity. Otherwise the equations $E = mc^2$ and $E = hv$ cannot immediately be combined into the wanted relationship.

The SI system is meant to develop in tandem with physics and chemistry, but in the move just presented the SI system seems to lay claim to be one step ahead. As far as I know, there is still no theory, superstring theory or overarching relativistic quantum-mechanical theory, which has managed to combine the relativistic mass of relativity theory with the rest mass of much of quantum mechanics and quantum chemistry. It seems as if the New SI metrologists were at least for a while trying to synthesize where physicists have not yet been able to do so.

However, in the specification that was presented in March 2013, the straightforward derivation above is forgotten, and instead the authors rely on de Broglie's hypothesis that not only particles that lack rest mass (photons) conform to the equation $E = hv$, but that even particles with rest mass do. There are not only radiation waves, there are also matter waves; waves whose frequency is proportional to the rest mass of the particle in question:

> Note that according to the theories of special relativity and quantum mechanics, an atomic particle of mass m, and hence from the Einstein relation of total energy $E = mc^2$, may be interpreted as a wave with an oscillation frequency, called the de Broglie-Compton frequency, given by $v = mc^2/h$. (DraftCh2_4March2013: 8)

I have explicitly been told that this is how the New SI authors today look at the relationship $m = hv/c^2$; the point being, that for a particle at rest the variable m represents only the rest mass of a particle, not a relativistic mass. In other words, the variable v used does not represent radiation frequencies but de Broglie-Compton frequencies. In my opinion, even so regarded the relationship is far away from being epistemologically on a par with the electrodynamic relationship used in the ampere definition, not to speak about the kinematic relationship behind the meter definition. I will make four remarks connected to $m = hv/c^2$; the first three are about v and the fourth about h. At least taken together, they ought to show that the new kilogram definition had today better be withdrawn; especially since there is another definition proposal that is not contaminated by similar obscurities.

I.

I start with a general rhetorical question: can the existence of matter waves really be regarded as so ascertained that they should be allowed to be an essential part of a kilogram definition? Isn't giving matter waves this kind of high epistemological status just another way to move forward faster than the physicists themselves are moving? Even in the quotation above there is some hesitancy. It is said that the mass of an atomic particle "*may be interpreted* [italics added] as a wave with an oscillation frequency." Shouldn't it from a metrological point of view rather be beyond all present reasonable doubt that all atoms, molecules, and macromolecules have an oscillation frequency?

II.

The equations used in the meter and the ampere definitions ($v = l/t$ and $q = i\ t$) contain three variables each, and only a specification of t is needed in order to obtain the needed direct mutual connection between the other two kind-of-quantity variables. In the formula $m = h\nu/c^2$, on the other hand, there are four kinds-of-quantities involved. That h and c are constants, not variables, does not change this fact.

As c and h appear in the formula, they are no more than the variables m and ν tied to any specific measurement units (see subsection 6a). They are, however, tied to specific kinds-of-quantity just as much as m and ν are. A fundamental constant is an unchanging magnitude of a certain kind-of-quantity. The constant c can be stated in meters-per-second, kilometers-per-hour, yards-per-minute, and many others, but all these units have to be units for velocity. Similarly, the Planck constant is a constant magnitude of the kind-of-quantity action, whatever standard unit it is ascribed; in the SI it is joules-times-seconds. As the symbol c represents a constant velocity magnitude, the symbol h represents a constant action magnitude. This being so, the question is: how does the New SI choose the specific frequency ν needed for the definition to work?

What we find stated is a relationship that can be rewritten thus: 1 (kg) = 1.475 521... × 10^{40} $h\nu_{Cs}/c^2$ (SI9 Draft 20013: 13). Here, ν_{Cs} represents a frequency (in hertz) of a radiation that occurs in certain energy level transitions in caesium 133 atoms, and which always has the same frequency;[20] h and c are the usual constants expressed in SI units. At first, this may seem unproblematic. What is to be regarded as 1 kg rest mass is defined by a mass multiple (1.475 521... × 10^{40}) of the three constancies of nature, h, c and ν_{Cs} as related in $h\nu_{Cs}/c^2$. On a second thought, however, an odd thing is at once revealed.

The frequency used (ν_{Cs}) is not a de Broglie-Compton frequency, but a radiation frequency. It seems as if the authors in spite of statements to the contrary have forgotten that they cannot without further ado combine the equations $E = mc^2$ and $E = h\nu$ into $m = h\nu/c^2$ and let m represent rest mass.

III.

If the authors had really trusted their view that the kilogram definition only relies on de Broglie-Compton frequencies, they could have reasoned much more straightforwardly. Let me for a moment play with $m = h\nu/c^2$ on the assumption that in fact all kinds of atoms, molecules, and macromolecules have a matter wave with a de Broglie-Compton frequency.

On this assumption it is possible to use de Broglie-Compton frequencies the way electric current is used in the present ampere definition. One may then speak of specific non-existing matter waves the way the ampere definition speaks of electric currents in non-existing conductors of infinite length. That is, one may then speak of a de Broglie-Compton frequency that is equal to the radiation frequency ν_{Cs}. One can simply substitute ν_{Cs} by the corresponding de Broglie-Compton frequency and so obtain a definition of a *rest mass magnitude* of 1 kg.

Even more, one can put in a frequency that directly corresponds to a rest mass of 1 kg. There is no longer any reason to put in a frequency that corresponds to a rest mass that is only a fraction of 1 kg. The meter definition can be formulated thus: 1 meter is the distance travelled in 1 second by light in vacuum when its velocity is given the value 299 792 458 m/s.

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[20] What I write as ν_{Cs} is in the draft symbolized by $\Delta\nu(^{133}Cs)_{hfs}$; hfs is short for hyperfine splitting, and Δ symbolizes that ν originates from an energy level difference.

A similar formulation of the content of the proposed kilogram definition looks like this: 1 kilogram is the rest mass of a particle whose matter wave frequency in hertz is such that the Planck constant is equal to $6.626\ 069\ 57 \times 10^{-34}$ J s.

If in the formula under discussion we put in the values for *c* and *h* in SI units, and put in 1 kg on the left hand side, then we can easily calculate a de Broglie-Compton definition frequency (ν_d) for 1 kg. That is, by means of the equation $1 = h\nu_d/c^2$ we can find the de Broglie-Compton frequency that shows that something has the rest mass 1 kg. The rounded number of ν_d is 1.4×10^{50} hertz. As far as I know, no physicist thinks that samples of molecules can be ascribed a de Broglie-Compton frequency. Otherwise this frequency could lay claim to be the de Broglie-Compton frequency of the kilogram prototype.

The purpose of these last remarks is to show, that if the New SI authors had found the existence of matter waves to be truly unproblematic, then in all probability they would themselves have formulated the views I have just played with.

What I have said so far in section 6a can be summarized thus: (i) if the New SI authors think they can in general combine the equations $E = mc^2$ and $E = h\nu$ into $m = h\nu/c^2$, then they are surely ahead of physics, and (ii) if they think, their own open hesitancy notwithstanding, that they can rely on a general acceptance of the existence of matter waves, then they are probably ahead of physics. Upshot: quite independently of what I will say in remark IV below, it seems today much too speculative to base the kilogram definition on the relationship $m = h\nu/c^2$.

IV.

The constant velocity magnitude *c* is the velocity of light in vacuum, but what can we say about the constant action magnitude *h*? It must be worthwhile to consider a little what kind-of-quantity the notion of action is tied to.

Ordinary physical and chemical kinds-of-quantity can in a commonsensical way be regarded as *properties* of particles, waves, statistical ensembles, movements, or processes.[21] Even though action might be called a property in a wide sense of this term, it cannot be a property in the ordinary sense in which the kinds-of-property length, mass, velocity, energy, electric charge, and (intensity of) electric current are properties.

The properties listed can all of them be ascribed to an entity at a certain point of time. Action, however, is defined as energy *multiplied by time*, which means that it must be thought of as extended in time. It is as impossible to think of an action as existing at a point of time, as it is impossible to think of a volume as existing in a plane, a surface existing in a line, or a line in a point. Therefore, the constancy of nature that is reflected in the notion of the Planck constant cannot be a constancy of the kind exemplified by *c* and some properties of atoms; this remark applies even to the pre-QM Planck constant.

This special feature of action is never mentioned and remarked upon in the New SI, but I think it is such a peculiar feature that until the nature of action has been better clarified, metrologists should be extremely reluctant to make the Planck constant central to a base unit definition. The concrete kilogram prototype can be substituted in other ways. The kilogram magnitude can be defined by the constant mass of a kind of atom. And one specific such proposal was already some years ago put forward as an alternative to the New SI definition:

> A kilogram is the mass of $84\ 446\ 889^3 \times 1000/12$ unbound atoms of carbon-12 at rest and in their ground state. (Hill, Miller, Censullo 2011: 84a)

[21] Of course, this is only the case when they are regarded as referring to something real, not when regarded as only calculation-simplifying mathematical tools. However, if action is only regarded as a mathematical concept, it cannot be regarded as an invariant of nature.

Here, the kilogram magnitude is a multiple of the mass of a specified kind of atom. In logical structure, the definition is completely analogous to the SI definition of the meter between 1960 and 1983, when the meter was defined as a multiple of a certain wavelength of a certain radiation.

Conclusions of Section 6: (i) in the next SI system the present distinction between base units and derived units ought to be cancelled, and instead only standard units that in various ways are dependent on each other should be spoken about; (ii) in the next SI system the kilogram definition proposal of Hill, Miller, and Censullo (or something similar) ought to be inserted, not that of the New SI.

7. The Case of the Mole and Counts – a Plea for the Natural Numbers

Of its base units the New SI says: "Of these definitions only the first (for the second), and the sixth (for the mole), are independent of the other definitions" (SI9 Draft 2013: 17). In this section I will discuss the mole, and in the next the second. In both cases I will criticize the SI system, both the present and the new proposal, for not having paid due attention to the distinction between, on the one hand, discrete entities and the natural numbers as a set of numbers outside the number line, and on the other continuous quantities and the whole number line; see (Strömdahl 1996: chs. 5–6),[22] (De Biévre 2007, 2011), (Price, De Biévre 2009), (Johansson 2011), and (Cooper, Humphry 2012). There is a reason why the mole and the second "are independent of the other definitions"; fundamentally, they rely on kinds-of-quantity that "have the nature of a count" (see quotation below).

Both the present and the New SI mention that there are kinds-of-quantity that are not taken care of by the SI system of units:

> There are also some quantities that cannot be described in terms of the seven base quantities of the SI at all, but have the nature of a count. Examples are number of molecules, [and others]. (SI8 2006: 105) (SI9 Draft 2013: 3)

The definition of the mole states: "When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles" (SI8 2006: 105). All of these, not only number of molecules, have the nature of a count, which means that without the use of any SI units at all, one can start to speak of number of atoms, ions, electrons, and other particles. Therefore, one may reasonably ask: for what reasons were once the mole unit and its kind-of-quantity (amount of substance; outside the SI sometimes also called chemical amount) introduced? Before trying to answer this question, I will outline how I think the mole had better be looked upon.

Physically useful mathematics need not have a direct relation to physical reality. For instance, there is a whole branch of mechanics, continuum mechanics, which consciously models the mechanical behavior of materials *as if* the materials in question are continuous substances, and not the unities of discrete particles such as atoms and molecules which they in fact are. Similarly, in my opinion, the continuous kind-of-quantity amount of substance is to be regarded as only an instrumentally useful fictional kind-of-quantity, not as a kind-of-quantity on a par with length and mass, which are understood as being real features of reality.

In areas of physics and chemistry that theorize about very large samples of objects, it is often mathematically much simpler to use continuous variables than discrete ones. The fact that the calculations now and then give results in decimals, which contradicts the view that the

[22] The first one to have seen the point clearly seems to be the German chemist Johann Weninger; for a summary of his views, and references to his writings between 1959 and 1990 (all in German), see (Strömdahl 1996: ch. 6).

object of investigation is constituted by discrete entities, is made practically unimportant by the large number of objects; it doesn't practically matter in what way the decimals are deleted.

Let us now, like the SI, use n as a continuous variable for moles of something, and N as a variable for entities that have the nature of a count, i.e., all specific values of N are natural numbers (SI8 2006: 115). Treating the Avogadro *number* (A_N) as some kind of conversion and/or scaling factor, we can stipulate this connection rule between the positive real numbers n and the natural numbers N:

$$n \text{ mol(X)} = (A_N N)\text{(X)}.$$

Here, "n moles of X" and "($A_N N$) number of Xs" always refer to the same sample of discrete entities, X. In more words the equality says: n moles (of a sample of entities of a specified kind) = the Avogadro number multiplied by the number N (the actual number of the entities in question).

The equality n mole = $A_N N$ is neither an ordinary conversion rule such as x yard = ($0.9144\,x$) meter nor an ordinary scaling rule such as x km = ($1000\,x$) meter. Note that these two rules for the continuous kind-of-quantity length have the same logical-mathematical structure. The same structure is also shared by scaling rules for discrete entities, e.g. x dozens of entities = ($12\,x$) entities; 3 dozens of egg = (12×3) eggs. In these kinds of rules the same variable (x, be it continuous or discrete) appears on both sides of the equality sign. In the connection rule under consideration, this is *not* so; the variable n is continuous, but N is discrete. Nonetheless a specific value of n shall correspond to a specific value N, and vice versa. How is this possible? As pointed out by Strömdahl in his discussion of the mole, it has been proven (by the famous mathematician Cantor) that a one-to-one mapping between the real and the natural numbers is logically impossible (Strömdahl 1996: 153). My solution to this connection puzzle looks as follows.

We can always without further ado move from right to left in the connection rule, i.e., map the natural number values of N onto the real number values of n. Since the real number line contains the natural numbers, we can simply make n equal to N. The structure of the connection rule n mole(X) = ($A_N N$)(X) is then exactly the same as in the pure scaling rule n dozen(X) = ($12\ N$)(X). We cannot, however, in the same straightforward way move from left to right in the rule. Since n is a real number, we cannot in decimal cases find the corresponding natural number N without a rounding or truncating rule that take the decimals away, but adequate such rules are, I take it, quite possible to construe. By using such rules the Cantorian proof mentioned can be bypassed, and a connection rule between n and N can be construed.

I hope this is enough to show that it is possible to treat the mole as a unit for a continuous but fictional kind-of-quantity (amount of substance, chemical amount), which is grounded in real discrete entities, and numerically connected to them by means of the connection rule presented.

Instead of in this way making the mole a very special kind of factor that connects with one another a continuous and a discrete variable, and of regarding amount of substance as only an instrumentally useful fictional kind-of-quantity, both the existing and the New SI present amount of substance as being a kind-of-quantity on a par with those of length and mass. As usual when people are logical, from the introduction of one oddity in a system there follow others. One logical consequence of treating the mole as an ordinary base unit for an ordinary kind-of-quantity is that the SI had to exchange the Avogadro *number* (A_N) for the Avogadro *constant* (N_A). Why?

In the connection rule introduced, n mole(X) = ($A_N N$)(X), the right hand side is ontologically more basic than the left hand side, since it is discrete entities or samples of such that are what modern chemistry is about. On the other hand, if the continuous mole variable *n* on the left hand side is regarded as representing the ontologically basic variable, and the mole as representing a true measurement unit, then, following the ordinary prescriptions of the SI system, there must be a measurement unit on the right hand side, too. And so the SI system constructors introduced one. The rule n mole(X) = A_N N(X) was replaced by n mole(X) = N(X)/N_A, where N_A, the Avogadro constant, is ascribed the measurement unit reciprocal mole, 1/mole; see (SI8 2006: 115).[23] Whereas the Avogadro number (A_N) is just a number, the Avogadro constant (N_A) is made to fit the generic symbolism for continuous kinds-of-quantity, $Q = \{Q\}[Q]$; see Section 6a. The constant is claimed to consist of both a number and a measurement unit: $N_A = \{A_N\}[1/\text{mole}]$.

Now I will present my two-fold reproach. First, the connection rule used in the SI, n mole(X) = N(X)/N_A, is put forward without any mentioning of the problem that *n* is a continuous and *N* a discrete variable. As Strömdahl puts it:

> There is an intrinsic qualitative difference between a variable isomorphic with the real number system and a variable isomorphic with the natural number system. [---] As long as there are doubts about the mathematical nature of the metric of *amount of substance*, it will continue to carry considerable ambiguity and indeterminacy. (Strömdahl 1996: 153 and 154)

Second reproach, since every base unit is base unit for a kind-of-quantity, even the reciprocal mole must be unit for a kind-of-quantity. But (like many others) I cannot understand what kind-of-quantity *reciprocal amount of substance* can possibly be. This means, in turn, that neither can I understand what kind of magnitude in the world the unit concept 1-per-mole (1/mole) is meant to be coordinated to. Traditionally, measurement units of the kind *x*-per-something (*x*-per-second, *x*-per-meter, *x*-per-mole, etc.) were taken to require a unit for a real kind-of-quantity as a value of *x*, and I think this requirement must be adhered to. That is, in my opinion kilogram-per-mole makes sense, but 1-per-mole does not.

Since the symbol A_N refers to a pure number, the connection rule n mole(X) = (A_N N)(X) looks like what it is, namely a simple conceptual rule. The formula n mole(X) = N(X)/N_A, on the other hand, contains the symbol N_A that is meant to refer to a magnitude, not to a pure number. Therefore, it looks almost like a natural law. And when it (wrongly) looks like a law, the Avogadro constant (wrongly) looks like a fundamental constant. However, the SI does for some good reason – never spelled out! – not call N_A a *fundamental* constant; it is called a *universal* constant (SI8 2006: 115). In the New SI the last term is dropped, and at first it seems as if the proposal really would like to call N_A a fundamental constant, but later on this is denied. To start with, the proposal says as follows about the constants that are meant to ground the base unit definitions:

> The seven defining constants […] are chosen from the fundamental constants of physics (broadly interpreted) that may be called constants of nature, because the values of these constants are regarded as invariants throughout time and space. (SI9 Draft 2013: 11)

However, despite being placed within a parenthesis, the expression “broadly interpreted” should be taken very seriously. Later the draft says:

[23] The rule is actually written n(X) = N(X)/N_A, but by inserting “mole” I am only making explicit the premise that *n* is a variable for moles.

> [T]he Avogadro constant N_A, ha[s] the *character of conversion factor* to convert […] the mole into the counting unit 1 for measurements of amount of substance [italics added]. (SI9 Draft 2013: 18)

A conversion factor is a pure number, but the Avogadro constant is not a pure number. As explained above, it is regarded as a number attached to a measurement unit. I guess this is the reason why the authors of the New SI say that it only has the "*character of* conversion factor." The really remarkable thing, however, is that something which has the character of a conversion factor is proposed to be a unit-grounding constancy of a base unit. The Avogadro constant, no more than the Avogadro number, can possibly be an invariant of nature.

It should also be noted, that at the moment when this paper was close to be finalized, there appeared a paper by strong proponents of the official proposal that anew treats the Avogadro constant as an ordinary fundamental constant (Milton et al. 2014).

Now I will return to my introductory question: for what reasons were once the mole unit and its kind-of-quantity (amount of substance) introduced? The mole is the base unit that was added only in 1971.

From a sociological point of view, it can of course be said that many metrologists might have had the all too common tendency to try to extend something good beyond its natural border, and that many chemists might have had a feeling that an incorporation of a chemistry unit in the SI system would enhance the status of chemistry. I think, however, that one should not *reduce* the incorporation to a sociological issue. There were also two theoretical issues involved; and solutions to theoretical issues are hardly ever self-evident. Let me add some very brief words about these issues.

First, the continuous variable n appeared in the gas law $p\ V = n\ R\ T$ before it was a settled issue whether gases are continuous substances or samples of discrete entities. When later it was settled, there arose the question how the gas law as traditionally conceived should be related to the similar law containing the Boltzmann constant, $p\ V = N\ k\ T$; see (Johansson 2011: sect. 4).

Second, the SI system has been, and is, intimately fused with the so-called *quantity calculus*. Some of my views above contradict parts of the quantity calculus as this is today used within the SI system. The system stipulates the existence of a certain dimension (something like a kind-of-quantity) called dimension one, and a standard unit for it called 1. For detailed criticism of the official present-day SI view of the quantity calculus, see (Emerson 2002, 2004a, 2004b, 2008) and (Johansson 2010: sect. 3–5).

The New SI wants the following definition of the mole:

> The mole, symbol mol, is the SI unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles; its magnitude is set by fixing the numerical value of the Avogadro constant to be exactly $6.022\ 141\ 29 \times 10^{23}$ when it is expressed in the SI unit mol^{-1}. (SI Draft 2013: 16)

From what I have claimed in this section, it follows that the notions of amount of substance, the Avogadro constant, and mol^{-1} are metrologically bad constructions. Therefore, I would like to end by proposing the definition below, which I know aligns well with proposals already made by a number of chemists (De Biévre 2013):

> The mole, symbol mol, is the SI unit for measurements of large numbers of elementary entities; the entities must be of the same specified kind. 1 mole consists of (the Avogadro number) $6.022\ 141\ 29 \times 10^{23}$ entities.

The expression "large numbers" makes it immediately clear that the mole unit is not intended for formulas such as $N_2 + 3H_2 \rightarrow 2NH_3$ when these are interpreted as being about individual molecules and not about moles of molecules. In the kind of formulas now mentioned, there are no variables for real numbers, only specific natural numbers.

8. The Case of the Second – the Most Basic New SI Base Unit

In the New SI the second has, in the way pointed out in Section 2, a more basic role than it has had before. It has never been defined by a prototype; until 1960 it was defined as the fraction 1/86 400 of a mean solar day, and between 1960 and 1967 it was defined as the fraction 1/31 556 925.9747 of the time duration of the Earth's orbit around the Sun in the year 1900. The possibility of building atomic clocks that served as a *mis en pratique* for the constancy of the radiation from some atoms changed the metrological agenda, and since 1967 the second is defined to be the time duration magnitude of 9 192 631 770 *periods* of a certain radiation (SI 2006: 113). I have no more to say about these definitions than what has already been said in the sections about conceptual and epistemic circularities.

The New SI, however, proposes this definition:

> The second, symbol s, is the SI unit of time; its magnitude is set by fixing the numerical value of the unperturbed ground state hyperfine splitting frequency of the caesium 133 atom to be exactly 9 192 631 770 when it is expressed in the SI unit s^{-1}, *which for periodic phenomena is equal to Hz* [italics added]. (SI9 Draft 2013:12)

This definition relies as much as the present on the assumption that the hyperfine splitting frequency of the caesium 133 atom is a spatiotemporal constancy.[24] However, *a difference is created by the move to make this definition conform to the logical structure of the other New SI definitions, i.e., always to let the unit-grounding constancy belong to another kind-of-quantity than the kind-of-quantity to which the explicitly defined unit belongs*.

In the cases of the meter, the ampere, and the kilogram, the SI definitions rely, I argued, on specific physical relationships, and I found the relationship used in the kilogram case to be highly dubious, but I did not question any of the kinds-of-quantity talked about. Here, in the case of the second, I find the very kind-of-quantity on which the proposed *new* definition is made to rely quite mysterious.

Every standard unit is a unit for some kind-of-quantity, as the second is the unit for time duration,[25] but what kind-of-quantity can frequency be when it is ascribed a reciprocal second ($s^{-1} = 1/s$) as its standard unit?

In Section 7, I pointed out that measurement units of the kind *x*-per-something require a unit for a real kind-of-quantity as a value of *x*, but in 1-per-second (just as in 1-per-mole) there is no such real unit. The part of the definition that I have italicized does not solve this problem. If the presumed unit 1-per-second is turned into the unit periods-per-second, i.e.,

[24] In quotation, the present definition is: "The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom" (SI8 2006: 113). It is then added: "This definition refers to a caesium atom at rest at a temperature of = 0 K" (ibid.)

[25] Note that *in itself* a single point of time can only be ascribed the duration 0 seconds (on the SI ratio scale for time); only the *duration* between a time point and a conventionally chosen zero point can be ascribed a non-zero number of seconds.

into hertz (Hz), then everything is as it traditionally used to be. Hertz is the unit for the kind-of-quantity *periods/cycles*-per-time-duration. What kind-of-quantity the unit reciprocal second is a unit for, remains a mystery. That it is given the name "frequency" is of no help. Rather, it is confusing; since traditionally frequency is periods/cycles-per-time-duration. Simply put, there is no invariant of nature that belongs to the curious presumed unit reciprocal second.

The ghost here is the dimension one and its standard unit 1, which I dismissed in the last section; and I will rest content with this dismissal even here. I leave it to the readers to try to make sense of the unit reciprocal second in the light of (Emerson 2002, 2004a, 2004b, 2008) and (Johansson 2010: sect. 3–5).

That there is something odd with ascribing frequency the unit 1/s can also be seen from the fact that the New SI proposal itself is not able always to stick to this view. Thirteen pages after the presentation of the definition of the second, it does repeat the view that the derived quantity frequency has the unit 1/s (SI9 Draft 2013: 25), but only one page later it then says: "The SI unit of frequency is given as the hertz, implying the unit cycles per second" (SI9 Draft 2013: 26) (SI 2006: 120). That is, the New SI manages to claim that the kind-of-quantity frequency has both the measurement unit 1-per-second and the unit periods/cycles-per-second.

The latter traditional unit is unproblematic, but it may look problematic to some SI metrologists. Why? Answer: because the kind-of-quantity cycle/period is neither a base nor a derived kind-of-quantity in the SI. How can it be outside the system? Answer: because it has the nature of a count, and what has the nature of a count is not in need of a measurement unit. In the former section, I claimed that the mole ought to be defined by means of elementary entities, i.e., by entities that have the nature of a count. Now I claim that the hertz should be defined as being the unit 1 cycles/periods-per-second, where cycle/period has the nature of a count. If this is accepted, the magnitude of the second can even in the future be defined by the time duration of a number of cycles/periods of a certain radiation.

In both the case of the mole and the new definition of the second, things go wrong because those who are in favor these definitions refuse to let entities that have the nature of a count to be basic parts of metrology.

My simple conclusion is that the present definition of the second ought to be retained as it is. Perhaps it could be added that the word "period" refers to phenomena that have the nature of a count, and that such kinds of entities do not need a standard unit. This is the reason why the definition of the second is independent of all the other base units.

9. The Possibility of Metrological Improvements

Is it possible to test whether or not an assumed unit-grounding constancy C_1 really is constant? Doesn't that require that we compare it with another assumed constancy C_2? And then we can of course ask how C_2 is to be tested; and so on into C_n and an infinite regress. Now, if C_1 and C_2 are nothing but spatiotemporally individual objects, then such an infinite regress does arise. However, as shown by the story of the kilogram prototype and its copies, not even a prototype is in fact treated as nothing but an individual object. It is in effect regarded as one of a number of exactly similar objects; and it can (at least in principle) be tested whether these particular objects change in relation to each other. If they do, something must be wrong somewhere. Thus: testing constancy need not involve an infinite regress.

When prototypes are exchanged for properties of atoms or electromagnetic radiation, then there is no longer any need to distinguish between prototype and copy. Each light ray in vacuum is so to speak both prototype and copy. Similarly, each caesium 133 atom is both prototype and copy; and the same goes for the elementary charge. Therefore, it is in principle possible to try to test whether their assumed constancy holds or not. However, in the New SI nothing is said about how to check the constancies it uses to ground the web of dependencies that exist between the units chosen. In this sense, it does take on the look of a Zanzibar system. The assumed constancy of the kilogram prototype was meant to be checked once in about every 40 years, even though after 1889 there were only two checks (1946 and 1989-91), but when and how are the presumed constancies of the New SI to be tested? No one knows.

There is also another but more indirect way in which lack of constancy can be detected. If the theory in which the constancy appears meets empirical anomalies, i.e., reliable measurements that do not fit what the theory predicts, then automatically the presumed constancy becomes questioned, too. For some months some years ago there seemed to be such a case.

In September 2011, physicists within the so-called Opera collaboration released results, which they claimed showed that neutrinos can move faster than light, but already in February 2012 they withdrew the claim. Now, if instead their initial beliefs would have been repeatedly confirmed, something would have had to be changed in relativity theory. Of course, whether or not the best change would be to say that *c* is after all not a constant is an open question. Nonetheless this "neutrino case" illustrates well the possibility of discovering that *c* is not a constancy of nature. And the same unspecific possibility exists with respect to *h*. Not even the inevitable epistemic circularities mentioned in Section 4 make physics and chemistry completely cut off from the possibility of meeting recalcitrant measurement data; data that later in retrospect may take on the appearance of falsifications.

Famously, in his *Philosophical Investigations* Ludwig Wittgenstein makes a metrological remark:

> There is *one* thing of which one can say neither that is one metre long, nor that it is not one metre long, and that is the standard meter in Paris.—But this is, of course, not to ascribe any extraordinary property to it, but only to mark its peculiar role in the language-game of measuring with a metre-rule. (Wittgenstein 1967: §50)

So far so good, one might say. Surely, nature had not given the standard meter the property one meter or any other numerical property. It was a community of human beings that by means of a coordinative definition ascribed the numerical concept "1 meter" to a property instance in Sèvre. But what Wittgenstein forgets is the fact that – very importantly – the standard meter was assumed not to change its length. And this is, I would say, to ascribe a macroscopic thing an "extraordinary property." Wittgenstein's philosophy contains no stress on the necessity of making changes in a language-game, but metrology needs changes now and then. Therefore, I would like to end by paraphrasing a famous passage from the philosopher of science Otto Neurath (the words within square quotes are his):

> [We] Metrologists are like sailors who on the open sea must reconstruct their ship but are never able to start afresh from the bottom. Where a [beam] unit definition is taken away a new one must at once be put there, and for this the rest of the [ship] metrological system is used as support. In this way, by using the old [beams and driftwood] units and new natural-scientific knowledge the [ship] metrological system can be shaped entirely anew, but only by gradual reconstruction. (Neurath 1973: 199)[26]

[26] Very early, Neurath subscribed to the views I have called inevitable conceptual and epistemic circularities, which only much later became the mainstream views in the philosophy of science. The quotation stems from

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Neurath's German book *Anti-Spengler*, 1921. In 1932/33 he re-used the metaphor, but in a shortened version that perhaps is more quoted (Neurath 1959: 201). The whole history of his metaphor is described in Thomas Uebel, "On Neurath's Boat" (Cartwright et al. 1996: part 2). Neurath was an ardent spokesman of logical positivism, but nonetheless he never shared two of the views that commonly are regarded as central to this anti-metaphysical movement: (i) that somewhere there are completely theory-free empirical data; (ii) that atomic sentences can be directly compared with reality. The solution to the puzzle is that Neurath, despite his holistic views, was of the opinion that a scientific and a naturalistic everyday language can be completely cut off from all metaphysics. Here I diverge from Neurath; I think, like Popper, that there is some overlap.

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