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LETTER TO THE EDITOR

On the structure of the New SI definitions of base units

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Abstract
This letter examines the structure of draft definitions of the New SI base units, which are posted on the BIPM website. It is argued that the new definitions of base units should be free from references to derived units and should not comprise conditional phrases; names of base units should be clearly distinguished from their symbols. To alleviate the identified problems, relevant modifications of the draft definitions are proposed.

1. Introduction

Although the ultimate responsibility for instituting the modernized International System of Units rests on the CGPM, the successful implementation of the New SI is a common objective for many organizations and individuals. The program is coordinated by the Consultative Committee for Units (CCU) and the information on current developments in this field can be found on the BIPM website (www.bipm.org), together with recent CCU reports [1, 2].

The draft Chapter 2 for the SI Brochure [3], posted on the BIPM website, presents proposed redefinitions of four base units of the New SI (the kilogram, the ampere, the kelvin and the mole) and offers rephrasing of the remaining three base units (the second, the metre and the candela); all of the new definitions have an analogous structure. The approach proposed in the Draft has been adopted in Resolution 1 of the 24th CGPM in October 2011, which provides a comprehensive, current official position on the way of instituting the New SI [4].

In this letter, I examine common elements of the New SI definitions of base units; the proposed new definition of the kilogram: '*The kilogram, **kg**, is the unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be equal to exactly* $6.626\,06X \times 10^{-34}$ ***when it is expressed in the unit*** $\boldsymbol{s^{-1}\,m^2\,kg}$***, which is equal to*** $\boldsymbol{J\,s}$.', serves as a representative example to focus attention on (the problem parts of the definition are indicated here in bold).

In the next section, I discuss the problem of derived units employed in the New SI definitions of base units. Next, I consider the use of conditional expressions in the definitions. The subsequent section deals with the distinction between the name of the base unit and its symbol. The identified problems can be solved by rewording of the Draft definitions. A brief summary and a discussion conclude this letter.

2. Definitions of the New SI base units should not involve references to derived units

Most definitions of the New SI base units have sequential character, i.e. they employ the previously defined base units; the only 'stand-alone' units (i.e. units that do not refer to any other base unit) are the mole and the second[1]. Moreover, most of those definitions make use of derived units, as summarized in table 1.

Definitions of base units should not employ derived units, because

(1) Derived units cannot be established *before* base units are defined; making use of the (previously undefined) derived units in definitions of base units violates coherence of the logical structure of the unit system. Opponents of the New SI should not be provided with the argument of circularity in the definitions.
(2) Derived units and base units should not be defined *simultaneously* (in the same definition), because that

[1] The New SI definition of the second does not actually require a reference to the kelvin; instead of referring to 0 K, the definition may explicitly state that the given numerical value of frequency ('splitting frequency') corresponds to the hyperfine transition in ^{133}Cs *in the absence of ambient radiation*. Section 2 shows that the New SI definition of the second does not have to employ the hertz either.

Table 1. Base and derived units, which are employed in the Draft definitions of the New SI base units.

#	New SI base unit defined	Base units employed	Derived units employed
1	s	K	Hz
2	m	s	—
3	kg	s, m	J
4	A	s	C
5	K	s, m, kg	J
6	mol	—	—
7	cd	s, m, kg	Hz, W, lm

(incorrectly) suggests that the units are, in principle, equivalent and the choice of the base unit might be discretionary. For example, the hertz (employed in the definition of the second) *cannot* be put on equal footing with the second, because the hertz is a unit valid only for periodic phenomena. Having admitted that the primary realization of the unit of time involves a periodic phenomenon and the hertz, we need to recognize that universality of the base unit (the second) cannot be narrowed by its particular *mise en pratique*[2]; the subordinate relation of the hertz to the second is irreversible.

(3) It is inconsistent to use the *symbol* of a (derived) unit that has not been previously named. The Draft definitions of base units employ symbols (Hz, lm, J, C and W) of derived units (the hertz, the lumen, the joule, the coulomb and the watt, respectively), which were not previously defined and *named*.

(4) Definitions of base units should not rely on implied knowledge of the user of the New SI. The Draft definitions of base units employ the hertz (Hz) and the lumen (lm) that require background information to be properly understood, and that information is not given in the definitions.

(5) Definitions ought to be as concise as possible and should not comprise unnecessary elements; definitions of the New SI base units are not the right place to introduce derived units.

(6) Definitions of the New SI base units constitute the *primary* reference for the entire system of units; therefore, the definitions *do not have to* conform to the tradition of using particular, convenient combinations of (derived) units in certain contexts, such as $[h] = \mathrm{J\,s}$ (rather than $[h] = \mathrm{s^{-1}\,m^2\,kg}$) in the definition of the kilogram, or $[k_\mathrm{B}] = \mathrm{J\,K^{-1}}$ (rather than $[k_\mathrm{B}] = \mathrm{s^{-2}\,m^2\,kg\,K^{-1}}$) in the definition of the kelvin. Logical consistency of the new, primary definitions must take precedence over tradition and convenience.

These problems can be solved by removing references to derived units from the New SI definitions of base units; it is sufficient to delete the phrase '***which is equal to*** (...)' from the definitions. If this suggestion is followed, the new definition of the kilogram will read: '*The kilogram,* ***kg****, is the unit of mass; its magnitude is set by fixing the numerical value of the Planck constant to be equal to exactly* $6.626\,06X \times 10^{-34}$ ***when it is expressed in the unit*** $\boldsymbol{s^{-1}\,m^2\,kg}$.'

3. Definitions of the New SI base units should avoid conditional phrases

Definitions of base units ought to have the form of positive rather than conditional statements, because

(1) The use of an alternative in a definition should be substantiated on logical or physical grounds; there is no logical or physical necessity to resort to alternatives in the New SI definitions of base units.

(2) The phrase '***when it is expressed in the unit*** (...)' suggests that there are other (equivalent) ways of expressing the unit; section 2 shows that the use of derived units in the New SI definitions is not a valid alternative.

(3) Definitions should be as concise as possible and therefore free from provisions that are not necessary.

The cure to this problem is to delete the conditional clause '***when it is expressed in the unit***', which can be done without loss of information. Then, the New SI definition of the kilogram takes the form: '*The kilogram,* ***kg****, is the unit of mass; its magnitude is set by fixing the value of the Planck constant to be equal to exactly* $6.626\,06X \times 10^{-34}$ $\boldsymbol{s^{-1}\,m^2\,kg}$.'

4. Definitions of the New SI base units should clearly distinguish between the name of the base unit and its symbol

If not only the name of a base unit, but also its symbol, is explicitly given in the definition (one might consider it optional), the unit and its symbol should be clearly distinguished for the following reasons:

(1) Definitions of base units ought to avoid any ambiguity and should not draw on implied knowledge of the New SI users. For example, although the difference between the unit of mass (the kilogram) and its symbol (kg) is evident for most people, it is not so clear in the case of the unit of amount of substance (the mole) and its symbol (mol).

(2) The situation where the name of the unit and its symbol are identical (yet not clearly distinguished in the definition) might be confusing: the name of the unit of amount of substance (the mole) and its symbol (mol) are different in English; however, the two terms are identical in quite a few languages, due to specific grammar and spelling rules, e.g. in Polish ('mol' and 'mol', respectively) or in

[2] Modern metrology draws on equations of advanced physics [5]; modern definitions of base units are abstract and have to be accompanied by *mises en pratique*. The New SI definition of the second in terms of the *atomic constant* implies a specific method of realization of the unit; this is a (currently unavoidable) weak spot of the definition, because other, newly developed methods can be more accurate. The other two 'mechanical' units (the metre and the kilogram) are defined by fixing the numerical values of *fundamental constants*, with no reference to their *mises en pratique*. Once the gravitational constant G is measured with relative uncertainty $\sim 10^{-8}$ this 'weak spot' of the definition of the second can be eliminated: all three 'mechanical' base units (s, m, kg) can be defined by fixing numerical values of three fundamental physical constants (c, h, G), whereas references to properties of material entities (atoms and elementary particles), physical phenomena and physical laws will occur exclusively in the *mises en pratique* of those units [6].

Russian ('моль' and 'моль', respectively, although the international symbol 'mol', written in the Latin alphabet, is also legal in Russia).

The solution to this problem is to add a phrase that explicitly identifies the symbol of a unit, e.g. '***denoted by the symbol***'. If this proposal is accepted, the New SI definition of the kilogram will read: '*The kilogram, **denoted by the symbol** kg, is the unit of mass; its magnitude is set by fixing the value of the Planck constant to be equal to exactly* $6.626\,06X \times 10^{-34}$ $\boldsymbol{s^{-1}\,m^2\,kg}$.'

5. Conclusion

Since the Draft definitions of the New SI base units are structured along the same pattern, they share common advantages and disadvantages. In this letter, I have discussed three sources of imperfections of the New SI definitions and the ways to resolve the identified problems; the discussion has been exemplified in the case of the New SI definition of the kilogram.

The same approach can be used to improve the other definitions of the New SI base units. For example, the Draft definition of the ampere: '*The ampere, **A**, is the unit of electric current; its magnitude is set by fixing the numerical value of the elementary charge to be equal to exactly* $1.602\,17X \times 10^{-19}$ ***when it is expressed in the unit s A, which is equal to C***.', can be modified along the lines discussed in this letter, which results in the following wording: '*The ampere, **denoted by the symbol A**, is the unit of electric current; its magnitude is set by fixing the value of the elementary charge to be equal to exactly* $1.602\,17X \times 10^{-19}$ $\boldsymbol{s\,A}$.', where the bold print indicates the amended phrases.

In addition to the common elements occurring in the Draft definitions, which were considered in this letter, there are also issues specific to individual base units, which are worth discussing separately, see, e.g., [7–10]. Although successful implementation of the New SI program hinges primarily on laboratory work, it seems prudent to encourage simultaneous discussion on phrasing of the New SI definitions of base units, so that consensus can be reached well before the CIPM recommendation is readied and the CGPM resolution on instituting the New SI is scheduled for voting.

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