An Alternative Path to a New SI

(Part 1. On Quantities With Dimension One)

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Abstract. Update of a system of units has always meant – and should mean – upgrade of the set of base units of the system of units. The forthcoming redefinition of the base units doesn't have such a purpose; therefore there are no grounds for speaking of a New SI. This article lists the changes in the set of base units, which – according to the author – can lead to a genuinely New SI.

1. What the Term "New SI" Should Mean

1.1 Redefinition of Units Doesn't Revise the Set of Base Units

The forthcoming redefinition of the base SI units [1] based on the fundamental physical constants (FPC) has agitated the whole world of metrologists and caused an avalanche of approvals, objections, corrections, and suggestions, touching upon not only the redefinition of units itself, but other aspects of metrology as well. The term "New SI" was coined.

But all the base SI units remain the same. What is being suggested is giving them new definitions that would enable to define the base unit values more accurately. Is revision of unit definitions a sign of a New SI?

Throughout the history of metrology a system of units was considered as *new* when the set of base units (system basis) of the system was changed. This happened in 1901, when CGS (the centimeter–gram–second system) was replaced with MKS (meter-kilogram-second), and later in 1935 – with MKSA (meter-kilogram-second-ampere). This also happened in 1954, when the MKSA system was extended to include new units and became known as meter-kilogram-second-ampere-kelvin-candela system; since 1960 this system is known as SI. Finally, a New SI could be spoken of in 1971, when SI got its seventh base unit, mole, the one most widely discussed today.

At the moment the situation is ambiguous. On the one hand, the base units of the SI are based on the base quantities [2, clause 1.10], while the ISQ on which SI is based [2, clause 1.6, note 2] has 7 base quantities [2, clause 1.4, note 1]. These base quantities are not going to be changed as a result of units redefinition. Therefore there are no grounds to speak of a New SI; rather, we can speak of the higher-quality, more accurate, more reliable SI.

Nevertheless, it is being said [2, clause 1.4, note 3]: "'Number of entities' can be regarded as a base quantity in any system of quantities.", presumably meaning ISQ as well. If this gets implemented, ISQ will have eight base quantities, SI will get the eighth base unit, and there will be grounds to speak of a New SI. This situation is described in article [3, section 5], but its author concludes: "to say that countable entities need a metrological unit 'one', is equally unwise as to say that the units of the base SI quantities should be tied up with the metrological unit 'one'". Hereafter, in section 3.2, we will express a different point of view.

The review article [4] devoted to history of the recent 50 years of the SI development, clearly shows well-grounded intentions of some metrologists to revise the base units, but not in the format that is currently being suggested. These intentions are supported with grounded assumptions.

Surely, the advocates of the forthcoming redefinition of the main SI quantities are well aware of all this, but consider revision of the base quantities ungrounded [5]. It's their right to do so, but in this case redefinition of the units cannot be considered as a sufficient ground to use the name ‘New SI’. Without a doubt, redefinition of units based on FPC is quite a useful and important activity, but it doesn’t constitute creation of a New SI.

1.2. On the Necessity of Revision of the ‘Dimension’ Definition

On the way to units redefinition an important event is happening in metrology: the dimension recording rules of derived units are being changed. For instance, in [1] the unit of electric charge is represented not as C = s·A, but as C = s A; the unit of energy is represented not as $J = m^2 \cdot kg/s^2$, but as $J = m^2\ kg\ s^{-2}$. This demonstrates negation of the metrological multiplication and division. The article [3] also states that "metrological multiplication doesn’t have any concrete physical or chemical sense".

The negation of metrological multiplication requires revision of the definition of dimension [2, clause 1.7]: "expression of the dependence of a **quantity** on the **base quantities** of a **system of quantities** as a product of powers of factors corresponding to the base quantities, omitting any numerical factor". It could be replaced, for instance, with such a definition:

"Dimension of a **base quantity** is represented by the symbol of this quantity defined by the standard. Dimension of a **derived quantity** is represented by consecutive record of the base quantities and their powers, not separated with spaces." It would be useful to include the following notes with this new definition:

1. "The dimension symbol of a base quantity is as a logical operator in the same way as ‘ln’, ‘sin’, ‘exp’, and therefore does not represent a quantity. It is possible to add that the dimension symbol shows the **kind** of quantity that is being defined [2, clause 1.2].

2. "The order in which the symbols of the dimension of a derived quantity are recorded is conventional and is defined by the standard."

1.3 Critical Analysis of the "Quantity of Dimension One" Concept

According to [2, clause 1.8], it is "**quantity** for which all the exponents of the factors corresponding to the **base quantities** in its **quantity dimension are zero**". "For historical reasons", according to note 1, it is also suggested to keep the term "**dimensionless quantity**". But if the exponents of the factors in the dimension of a quantity are zero, this doesn’t necessarily mean it has no dimension at all. That is why [2] prefers the term "**quantity of dimension one**".

This term includes several types of quantities that are completely different in nature, and it would be expedient to assign dimension to some of them. So the term "dimensionless quantity" shouldn’t be kept in use in metrology; its use should be limited to the works on history of physics and metrology. In the same way, the term "dimensional quantity" is to be left for historical use only, since every quantity should have a dimension.

Let’s pay attention to Note 3 of [2, clause 1.8]: "Some quantities of dimension one are defined as the ratios of two quantities of the same kind." The word "some" implies that not all of these quantities can be "defined as the ratios of two quantities of the same kind." Meanwhile, Note 4 says: "The number of objects is a quantity of dimension one". Therefore the Notes 3 and 4 prove that the name ‘quantity of dimension one’ combines at least two different concepts.

The definition of ‘quantity of dimension one’ mentions ‘factors’ in the quantity dimension. But since metrological multiplication is not accepted as legitimate, the very definition of ‘quantity of dimension one’ should be corrected. For instance, we can suggest the following definition: "the quantity, which, being a factor of any **quantity equation**, doesn’t change the dimension of the quantity defined by that equation."

The article [4] distinguishes 4 groups of quantities of dimension one.

<u>Group One</u> representing **similarity criteria** is analyzed in section 2 of this article. The article [3] contains the idea that ‘dimensionless quantities’ should be rather called ‘unitless quantities’. But section 2 will show that similarity criteria have off-system units.

<u>Group Two</u> includes quantities that can be united under the term "**number of entities**". The following is said about them in [2, clause 1.4, note 3]: "‘Number of entities’ can be regarded as

a base quantity in any system of quantities." Hence we can make a conclusion that ‘number of entities’ can have its own dimension and units. They will be discussed in section 3 of this article.

Group Three includes quantities describing rotation. SI has a system unit for some of them – it is radian. They will be discussed in section 4 of this article. However, the quantities describing rotation are included into a bigger group, "**cyclic quantities**", which also includes quantities describing oscillations and waves; they will be discussed in section 5 of this article.

Group Four includes logarithmic relationships that have their own units (e.g., decibel, neper). There is no rational solution suggested for them yet. That’s why they shouldn’t be included into classification of quantities of dimension one.

Thereby, the concept of "quantity of dimension one" includes 3 essentially different groups of quantities, one of which is divided into 3 subgroups (see fig. 1). Their detailed analysis allows to truly update SI.

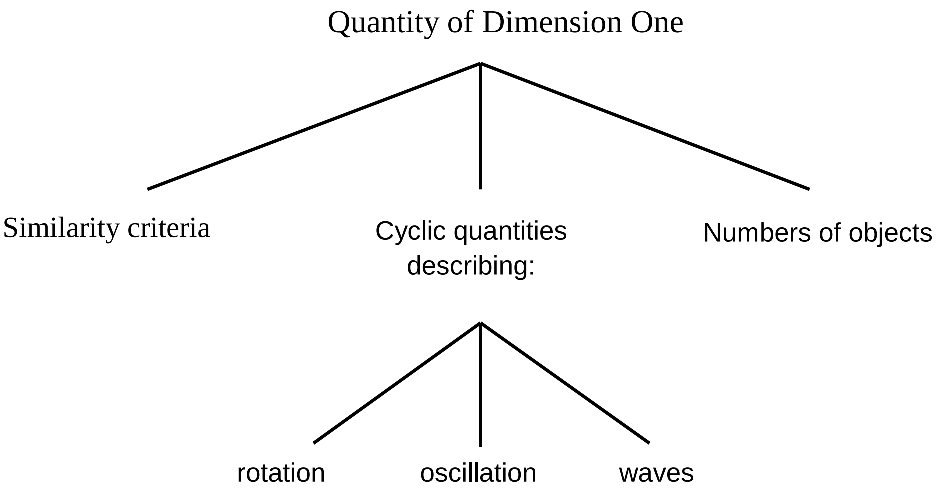

Fig. 1. The hierarchical chart of quantity of dimension one

2. The Content of the Concept ‘Similarity Criteria’

2.1. Quantity Equation that Defines Similarity Criterion

Analysis of definition of the measurement process shows the following. According to [2, clause 2.1] the term ‘**measurement**’ is defined as the "process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity." There is also Note 2: "Measurement implies comparison of quantities or counting of entities."

If we take into account the fact that dimension of a quantity is at the same time the dimension of its unit, we can see that dimension appears only when there is a unit with its dimension recorded beside the numerical value. This means a unit has to be used with a ‘dimensional quantity’, but doesn’t have to be used with a ‘dimensionless quantity’.

Let’s take a look at the following metrological equation [6, 3]

$$Q = \{Q\}\ [Q], \qquad (2.1)$$

where Q – analyzed quantity, $\{Q\}$ – its numeric value, $[Q]$ – unit of the quantity. Similarity criterion is the numeric value $\{Q\}$, while the unit $[Q]$ is the **basis** of the similarity criterion. Dimension of the basis defines dimension of the similarity criterion. If $[Q]$ is an **system unit**, then Q is usually called a "dimensional quantity", since Q is connected with a given system of units. If $[Q]$ is an **off-system unit**, then the numeric value $\{Q\}$ becomes a sort of ‘dimensionless quantity’, since it is not advisable to use off-system units in the SI.

If we rewrite the equation for similarity criterion $\{Q\}$ as

$$\{Q\} = Q\ [Q]^{-1}\ , \qquad (2.2)$$

it becomes clear that $\{Q\}$ is a 'quantity of dimension one' in any case, since the similarity criterion Q and its basis $[Q]$ have identical dimensions. Therefore, if the unit $[Q]$ is an off-system one, this doesn't necessarily mean that the quantity Q doesn't have a unit. At the same time, the lack of basis $[Q]$ is also unacceptable, because in this case the equation (2.2) looses its sense.

Let's give practical examples.

2.2. The Distinctive Feature of Similarity Criteria

Example 1. Let's compare the equation (2.1) with the formula for volume of the combustion chamber V above the combustion engine piston:

$$V = \varepsilon\ V_k\ , \qquad (2.3)$$

where V_k – is the volume of the combustion chamber when the piston is in its upmost position; ε – compression ratio. If we agree to consider V_k as an off-system unit in this field of technology, then equations (2.1) and (2.3) become identical. The content of compression ratio ε becomes non-different from the similarity criterion $\{Q\}$ in (2.1). The off-system unit for the volume of combustion chamber V_k has dimension of volume.

Example 2. Let's write down the quantity equation for velocity of airplane v:

$$v = M\ v_s\ , \qquad (2.4)$$

where v_s – velocity of sound in the atmosphere; M – Mach number. Let's assume that the airplane is flying in the air at temperature 0ºC and air pressure 1 atm at a speed of 662 m/s. In this case Mach number equals 2. The velocity of sound v_s can be considered as a off-system unit of aerodynamics. Mach number M then becomes non-different from a similarity criterion, which it in fact is. It is no coincidence that in this case aviation specialists would say that the airplane's velocity equals two Mach numbers. The off-system unit v_s for velocity of sound has dimension of velocity.

Example 3. In atomic physics a particle's intrinsic moment of momentum L, which is also called *spin*, is described with the quantity equation $L = \hbar J$, where $\hbar = h/2\pi$ – reduced Planck constant, h – Planck constant, J – spin quantum number. Physicists usually call both the "dimensional quantity" L and the "dimensionless quantity" J by the same name – *spin*, though two essentially different quantities shouldn't be called by the same name. Therefore it is correct to write the quantity equation as

$$L = J\,\hbar\ . \qquad (2.5)$$

In this example spin quantum number J is a similarity criterion, and the off-system unit of the reduced Planck constant $\hbar$ has dimension of action in the SI. For more detailed discussion of this, see section 3.6.

All the three similarity criteria (ε, M and J) from these three examples have dimension one, while each dimensional quantity (V_k, v_s and $\hbar$) is an off-system unit. And each of these off-system units has dimension different from 1.

Surely, if we take into account the huge number of different similarity criteria, each of which can have its own off-system unit, we can easily imagine how anxious the metrologists can get because of the huge number of off-system units. But the off-system units similar to the ones shown above really exist in practice, though they are not called by this name.

Off-system units can correspond to quantities with different physical content. For instance, reduced Planck constant $\hbar$ is a fundamental physical constant, V_k has fixed value only for a given type of combustion engines, and the value of velocity of sound v_s depends on characteristics of the air at the point where the airplane is. Spin quantum number J can be either an integer or a half-integer number, while V_k and v_s are noninteger numbers. It is quite acceptable to use phrases like “1.5 volumes of combustion chamber V_k” or “3/4 of the sound velocity v_s.” Many similarity criteria use variable quantities as their basis. For instance, the basis of the Reynolds number is friction force of the liquid on the wall, which depends on many factors.

Coclusion: similarity criterion is ***a ratio of a dimensional quantity to its basis, which has the same dimension and unit as the dimensional quantity, and is an system or an off-system unit of the dimensional quantity***.

2.3. An Option of Recording Dimensions and Units for Similarity Criteria

The physical content of a criterion can be derived only from its own quantity equation (if it is provided), and – indirectly – from the name of the similarity criterion. But the name is at best a verbal formulation; to a certain extent it is merely a convention. Is it possible to reflect the physical content of a similarity criterion by means of dimensions and units?

The author of the article [7] tried to follow the dimension definition (see Section 1.2) literally. As a result, he started recording dimension of a similarity criterion as the dimension of its basis raised to the zero power (zero is permissible as a power of dimension). Then, for instance, the dimension of relative linear deformation becomes L^0, and its unit is m^0; dimension of the Mach number equals $(LT^{-1})^0$, and its unit is $(m\ s^{-1})^0$. Such a notation immediately makes the differences between similarity criteria visible. This suggestion can be especially useful for teaching hydraulics and heat engineering, where many different similarity criteria are used.

There is only one requirement for this option: we should take into account the quantity equation that is initial from the point of view of the quantity content, rather than the form of the quantity equation that was obtained after cancellation of several quantities in the numerator and denominator of the quantity equation. For instance, as the dimension of the Reynolds number we should use not the dimension of dynamic viscosity η (based on the popular formula $Re = ud\rho/\eta$), but the dimension of force, since the Reynolds number is the ratio of inertial forces to viscous forces. This means, the dimension of the Reynolds number in the SI can be written as $(LMT^{-2})^0$, and its unit as N^0.

2.4. Unsystematic Naming of Similarity Criteria and Its Reasons

Similarity criteria have a number of different names in physics and technology, for instance, **ratio** (compression ratio, friction ratio, transmission ratio), **number** (Mach number, spin quantum number), or **coefficient**, though each of these terms has a different sense and content in mathematics.

Different kinds of similarity criteria can be found in every mathematical and technical discipline. Scientists were using them long before the theory of similarities itself was formed. And this is what caused such unsystematic naming.

For instance, in similar triangles the trigonometric functions of all the angles are criteria of geometrical similarity. The most ancient similarity criterion in mathematics is π – the relation between the circumference and its diameter. It is more correct to compare the circumference to its radius. However, this similarity criterion was devised so that it was convenient to calculate, because diameter can be measured much more conveniently and precisely than radius.

We have to accept that this is exactly how science was progressing: the obvious, convenient similarity criteria were chosen and fixed, and they were named the way their discoverers named them. Over time, this made teaching and learning science more complicated. As a result, methodologists of science have to face a collection of practicalities and inaccuracies piled up over centuries, and sooner or later they it has to be disposed of.

3. The Content of the Concept ‘Number of Entities’

3.1. The Distinctive Feature of the "Number of Entities" Concept

Practically in every branch of physics one can find a countable quantity with the following content: **number of entities** of homogenous system. The unit of this quantity has numerical value that equals to a positive integer. Universality of this quantity is discussed in the article [8]. This quantity does not depend on any other quantities. It doesn’t have a quantity equation. Back in the SI8 it was suggested to treat the number of entities as the base physical quantity in any system of units, and include it into the base quantities of the SI, which is also reflected in [2, clause1.4, note 3]. We suggest to use N as the symbol of this quantity; the name of its unit will be discussed in the end of this section.

Below is review of applications of the numbers of entities, based on the article [9], where the number of entities is named as the number of structural elements, the conventional term of the Russian standard concerning amount of substance.

The fact that the number of entities can be only integer doesn’t mean that the counted entities cannot be divided into parts. But in this case any part becomes an entity of another system, which is on a different level of state of matter. For instance, the entities of which gas is comprised are molecules. At high temperatures the molecules of the gas ionize, and the products of molecular decomposition become parts of plasma, that is, a different state of matter. Let’s give examples of the concept "number of entities" in different cases.

3.2. Number of Entities in Molecular Physics

One of the system units of the SI is **amount of substance**. According to the definition, “Amount of substance n is a physical quantity that measures the size of an ensemble of elementary entities, such as atoms, molecules, electrons, and other particles”. In the SI this quantity has dimension N; its unit is mole. It is defined by the equation

$$n = N / N_A , \qquad (3.1)$$

where N is the number of objects in a homogenous system; N_A – Avogadro constant; its unit is mole^{-1}. Numeric value of the Avogadro constant is called the Avogadro number A_N. It equals the number of atoms in 0,012 kg of the carbon isotope ^{12}C.

According to the definition [2, clause 1.4] a base quantity is "a quantity in a conventionally chosen subset of a given system of quantities, where no subset quantity can be expressed in terms of the others". According to the equation (3.1), amount of substance n depends on the number of entities N and the Avogadro number N_A. Thus, assigning amount of substance n as the base quantity contradicts the definition of the base quantity. The reason of this alogism is hidden in the two words from the definition of the base value: ‘conventionally chosen’.

Many chemists and metrologists disagree with the unit of the Avogadro constant (mole^{-1}). This is well described in the article [10]: "I cannot understand which natural quantity can have a unit 1 per mole (1/mole) and what it conforms to. Traditionally, measurement units of the kind x per something (x per second, x per meter, x per mole etc.) are measurement units of real quantities, and I think this requirement should be met. In other words, to my mind kilogram per mole has sense, and 1 per mole doesn’t." A review of the numerous critical remarks concerning the mole unit can be found in the article [4, problem 6].

The unit 1 per mole doesn’t follow unambiguously from the equation (3.1). If the number of entities N had its own unit (for instance, piece), it would follow from the equation (3.1) that the Avogadro constant has a unit pieces per mole, which has clear physical sense. This is how the introduction of the number of entities as a base unit of the ISQ can remove an existing alogism.

The same problem can be solved in a totally different way, by conversion of the equation (3.1) into the quantity equation

$$n_A = N / A_N \ , \qquad (3.2)$$

where the Avogadro number A_N will have dimension N and unit *piece*. In this case the amount of substance will be defined by the similarity criterion n_A, the numeric value of which will be 1 if the number of entities $N = A_N$. In this case, there will be no need in the mole unit at all. Chemists should decide if such a solution is suitable for them. But it would surely make metrologists' life much easier.

3.3. Number of Entities in Periodic Processes

Let's consider a quantizable periodical process, that is, a process consisting only of an integer number of oscillation periods. In such a process each specific oscillation period is an elementary entity of a periodical process. Surely, such representation of periodical processes is artificial, since a periodic process is continuous, and each period can be divided into fractions (has phase). Nevertheless, it is used quite often, especially for high frequencies.

In this approach the 'number of periods' can be considered as a particular case of the 'number of entities'. The same can be said about wave propagation and the 'number of waves.'

3.4. Number of Entities in Informatics

Introduction of the number of entities as a base quantity can help to solve one more problem that causes lots of discussions: is amount of information a quantity?

The amount of information usually stands for measure of information in a message. This quantity has its own unit in the informatics – it is called bit and defined as a unit of the amount of information in the binary computing, the minimal unit of the amount of information that can be transmitted or stored, and corresponds to one binary digit that can have one of the two values, 0 or 1.

Each bit can be represented with energy state of a technical device (trigger), which saves or transmits information. The fact that the trigger output has only two possible options of the number of entities (0 or 1), rather than, for instance, 10 as in the decimal system, doesn't make any essential difference. What differs the amount of information as a number of entities is the fact that its unit has numeric value in the binary system.

There is one more unit used in informatics; it is byte, and it equals 8 bits. It is the smallest addressable data unit in the memory of a computing machine. In this case octal system is used instead of binary.

3.5. Number of Entities in Economics

'Goods' is one of the main concepts of economics, and 'quantity of goods' is one of the main economic quantities. In most cases this economic quantity is measured in one of the three ways: in units of volume (cubic meter, liter, barrel, ounce, etc.), in units of weight (kilogram-force, ton-force, pound-force, etc.) and in pieces. As the packaging technologies evolve and gain increasingly wider use in loading, transportation, storage, and sales, and as a result of widespread use of containers, piece becomes the most popular unit for quantity of goods.

3.6. Number of Entities in Quantum Mechanics

As number of entities is introduced as a base quantity, in quantum mechanics this results in changing dimension and unit of the physical constant known as the Planck constant and denoted as *h*. The Planck constant is a quantum of the physical quantity 'action'; its content is defined with the following equation:

$$h = \varepsilon / \nu \ , \qquad (3.3)$$

where ε – energy of one quantum of electromagnetic radiation; v – frequency of quanta radiation. Based on equation (3.3), the Planck constant can be seen as amount of energy per unit of radiation frequency.

In the SI units, the Planck constant is expressed in joule seconds (J s). This unit is derived from equation (3.3) if we assume that the unit of frequency is 1 per second (s^{-1}) – though such a unit wouldn't have more sense than $mole^{-1}$.

In the quantum mechanics the word 'quantum' can be rationally used to denote the unit of the number of entities. Wikipedia gives the following definition of quantum: "the minimum amount of any physical entity involved in an interaction." If we use quanta per second as a unit of frequency v, dimension analysis of the equation (3.3) shows that the unit of Planck constant is (joule s $quantum^{-1}$).

Using SI unit doesn't allow to conclude that ε is quantity of energy of one quantum. This conclusion becomes possible only if we use a different unit for frequency v: (quantum s^{-1}) and a different unit for Planck constant h (joule s $quantum^{-1}$). The existing unit (joule s) is legitimate only we replace the energy of one quantum ε with the energy of n quanta ε_n, measured in the unit (joule quantum) in the equation (3.3).

If the reduced Planck constant $\hbar = h/2\pi$ is being used, it has a quantity equation different from equation (3.3):

$$\hbar = \varepsilon/\omega_0 \,, \qquad (3.4)$$

where ω_0 – angular velocity of rotation of radius vector on the coordinate plane in the method of vector diagrams. Let's note that the word 'reduced' is often omitted in the literature, so when Planck constant is mentioned, it's often not clear whether they mean h or $\hbar$.

Based on equation (3.4), $\hbar$ can be understood as quantity of energy per unit of angular velocity of the radius vector. But this explanation doesn't tell anything about the physical content of $\hbar$, since ω_0 a mathematical quantity that's artificially introduced into physics. Quantum mechanics discusses radiation, and not the unidirectional rotation of the radius vector that can be characterized with its angular velocity. The constant $\hbar$ is a mathematical interpretation of the Planck constant h – but it's mathematics rather than physics.

Any angular velocity should have unit revolutions per second or radian per second. In order for the equation (3.4) to comply with the dimension rule, we should assign the unit (revolutions $quantum^{-1}$) to the factor 2π in the expression $\hbar = h/2\pi$. This is how one ambiguity calls for another one.

In the SI $\hbar$ has the same unit as h. This can be one more reason why $\hbar$ is often called Planck constant. Only the absence of the units for the number of entities and the angular rotation in the SI allows the expression $\hbar = h/2\pi$ to comply with the dimension rule. At the same time, it creates terminological confusion.

3.4. The Usefulness of New Units by Example of Photoelectric Effect Description

An example of using new units is given in the paper [11, p.p. 168-171], where in the description of photoelectric effect the units joule per photon and joule per electron are used to measure energy per one particle. (De facto, in both cases the same unit is used – joule per quantum.) A. Einstein described energy conservation law of photoelectric effect with the following equation:

$$E_k = hv - E_i \,, \qquad (3.5)$$

which in [11] is written the following way

$$E_k = \hbar\omega_0 - \Phi, \qquad (3.6)$$

where E_k – a photon's kinetic energy ; $h\nu = \hbar\omega_0$ – photon's energy; E_i – an atom's ionization energy; Φ – electronic work function.

The author of [11] discovered that the summands of the equation (3.6) have different units: the units of E_k and Φ pertain to one electron, and the unit $\hbar\omega_0$ – to one photon. That is why in order to comply with the rule of dimensions, $\hbar\omega_0$, interpreted as a photon's energy, should be supplemented with a factor Y^{-1}, the sense of which is the ratio of the emitted electrons to the number of absorbed photons; its unit is electrons per photon.

De facto, there are two quantities missing in the equations (3.5) and (3.6): the number of electrons N_{el} and the number of photons N_{ph}. If they are introduced, the photoelectric effect equation will look as follows:

$$N_{el}\, E_k = N_{ph}\, \hbar\omega_0 - N_{el}\, E_i\,. \qquad (3.7)$$

Since N_{el} and N_{ph} have identical dimensions, the equation (3.7) sustains dimension analysis check. As for the factor $Y = N_{el} / N_{ph}$ known as quantum efficiency, it is a similarity criterion, and it can be included into equation (3.7) if we divide both parts of it by N_{ph}. In that case, the photoelectric effect equation will look as follows:

$$Y\, E_k = \hbar\omega_0 - Y\, E_i\,, \qquad (3.8)$$

which describes the physical content of photoelectric effect better than (3.6).

3.7. On the Name for a Unit of the Number of Entities

The question of naming a unit of the number of entities still remains unsolved. The author of [8] shared opinion that number 1 should be seen as a SI unit for the "numbers of homogenous elements," being aware that this would lead to update of the SI. He suggested to use symbol *I* and name *heis* for this unit (from the classic Greek εισ – one); at the same time he mentions that such a "unit" is integer in quantum mechanics only.

As it is shown in this section, different names are used for numbers of entities. The name 'quantum' connects the unit of the number of entities to quantum mechanics; the names 'period' and 'wave' – to the theory of periodic processes; the names 'bit' and 'byte' – to informatics.

The general, featureless term is 'piece', or *Stück* in German. In the linguistics this word stands for a separate object out of many homogenous countable objects. There is a word with similar meaning in every major European language.

Name of the unit for the number of entities is a matter of international convention. And while there is no convention on this subject, the unit for the number of entities is called its habitual names. There is even abridged symbolic record for the "piece" unit; for instance, pieces per cubic meter can be recorded as pcs m^{-3}.

4. Rotation Angle as a Base Physical Quantity

4.1. Classification of Movement Forms

The correct understanding of the basic concepts of "angular displacement" and "rotation angle" depends on classification of the body's movement modes (see Fig. 2).

Let's emphasize: we mean movement of a body, not movement of a point mass. Point mass is a mathematical concept, it is not compatible with real movement. If the size of the body is sufficiently small, the term 'elementary particle' is used. From the mathematical point of view this means that the particle's size should be infinitesimal. It can be so from the point of view of macro world, but not the micro world, where the sizes of particle can differ significantly, to say nothing of difference in their other properties.

In the general case, the displacement vector d**r** of the body's center of mass is considered as the coordinate of state of the moving body. **Displacement** is a vector connecting the final and initial positions of the center of mass of a moving body. But it's not the displacement that defines the movement form, because the coordinates of state are different for different movement forms. Classification of the body's movement forms helps to clarify their coordinates of state.

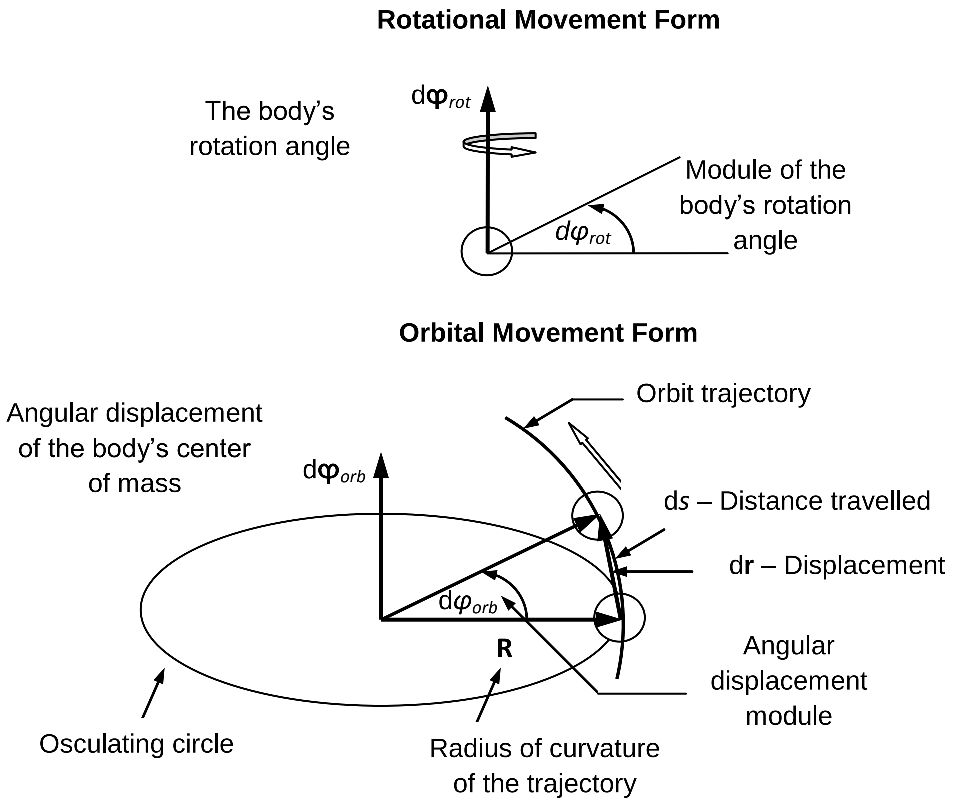

Fig. 2. Body Movement Forms

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1. Straight-line movement form. Its coordinate of state is **linear displacement** of the body's center of mass. This movement form also allows spinning movement of the body.

2. Rotational movement form. Its coordinate of state is pseudo vector of the **rotation angle** d$\boldsymbol{\varphi}_{rot}$ of a rotating body. This movement form pertains to the body as a whole, no matter if its center of mass is moving or not. Movement of the body's particles, which are not on the rotation axis, is not taken into consideration.

3. Orbital movement form. It considers movement of the body's center of mass along a curved trajectory with radius of curvature **R**. In the orbital movement form the coordinate of state is the pseudo vector of **angular displacement** d$\boldsymbol{\varphi}_{orb}$ of the center of mass of a moving body. In this movement form the body doesn't not necessarily have a spinning movement. In the general case, the orbital movement form consists of 4 different movement forms: spinning movement of the body, rotation of the radius of curvature around the center of curvature of the trajectory, and the two straight-line movements of the body's center of mass (along the radius of curvature and perpendicular to it).

4.2. What Is the Difference Between the Rotation Angle and the Angular Displacement?

The term 'displacement' is not acceptable for description of rotational movement, because 'displacement' means change of place. During rotation the body does not change its place; its rotation is described by the body's rotation angle $\boldsymbol{\varphi}_{rot}$. But its particles that are not on the rotation axis do change their places; their movement is described by angular displacement $\boldsymbol{\varphi}_{orb.}$ That's why rotation angle is a separate pseudo vector quantity, not the module of angular displacement. There is only one similarity between these quantities: both of them are measured in the units of plane angle; in the SI they are considered as 'dimensionless'.

In the same way, there is essential difference between the angular velocity of body's rotation $\boldsymbol{\omega}_{rot} = \mathrm{d}\boldsymbol{\varphi}_{rot}/\mathrm{d}t$ and the angular velocity of rotation of radius **R** of the trajectory of the body's center of mass $\boldsymbol{\omega}_{orb} = \mathrm{d}\boldsymbol{\varphi}_{orb}/\mathrm{d}t$. For instance, in the Solar system angular velocities of the planet's own spinning movement are considerably different from angular velocity of rotation of their centers of masses around the Sun. Moreover, angular velocity $\boldsymbol{\omega}_{rot}$ of any planet is constant, while angular velocity $\boldsymbol{\omega}_{orb}$ of the same planet's movement around the Sun is variable within each orbital cycle.

As for the angular displacement, there is opinion that it's not a vector, since it doesn't comply with the commutative rule. But angular displacement is not a true vector – it's an axial vector, or a pseudo vector.

Virtually all the scientific reality uses concepts "angular displacement" and "rotation angle". Therefore it is very important to have everything concerning these quantities strictly defined and grounded.

4.3. The Distinctive Features of the 'Displacement' and the 'Distance' Concepts

The 'displacement' and the 'distance' concepts are used for quantitative assessment of movement along a curved trajectory (see Fig. 2). The displacement $d\mathbf{r}$ in orbital movement is a chord subtending an arc or a curved trajectory. The distance $\mathrm{d}s$ is defined as the length of the trajectory, traversed by the center of mass. It allows to calculate dissipative energy losses during the body's movement. When the body moves along a closed trajectory, it's the distance, not the displacement, in which we are interested, since in the end of an orbital cycle the displacement equals zero, while the value of distance increases with each coming cycle.

Displacement $d\mathbf{r}$ in orbital movement is a vector product

$$\mathrm{d}\mathbf{r} = [\mathrm{d}\boldsymbol{\varphi}_{orb}\,\mathbf{R}]. \qquad (4.1)$$

While the orbit displacement is the distance $\mathrm{d}s$, defined by the scalar product

$$\mathrm{d}s = \mathbf{R}\,\mathrm{d}\boldsymbol{\varphi}_{orb}. \qquad (4.2)$$

If we use the SI units, it follows from the equation (4.2) that the unit of path $\mathrm{d}s$ (meter) does not correspond to the unit of the product $\mathbf{R}\mathrm{d}\boldsymbol{\varphi}_{orb}$ (m rad). This discrepancy was already pointed at in the article [7], where it was suggested to consider the unit of the radius of curvature **R** equal m rev^{-1} (in the SI – the unit m rad^{-1}). The same point of view was already published earlier, in the articles [12, 13]. It is not understandable, why the unit radian is being lost during unit analysis of the equation (4.2).

The unit of the radius of curvature **R** that equals m rev^{-1} is quite an unusual conclusion for physics and metrology, but it doesn't lead to breaking the dimension rule. On the contrary, it leads to the unit of the trajectory curvature that equals rev m^{-1} (in the SI – rad m^{-1}) instead of the unit currently used in the SI, m^{-1} (inverse meter).

4.4. What Is the Difference Between the Rotation Angle in Physics and a Plane Angle?

There is a very peculiar situation in literature on physics: the angular displacement and the rotation angle are often defined as scalar quantities, while their time derivative (angular velocity) is always defined as a vector quantity. The reason of this discrepancy is probably the fact that both the angular velocity and the rotation angle are assessed in the plane angle units.

But the plane angle is a geometric figure, and its value is a *mathematical quantity*, while rotation angle is a *physical quantity*. This is the most general definition of a plane angle in geometry [14]: "figure formed by two rays or segments, which are called *sides* of an angle, that have a common end point, which is called the *vertex* of the angle".

Angular displacement is assessed not as a physical quantity but as a geometrical one, i.e. in the units of the plane angle, by which a dot or a line rotates against around a fixed axis, that is, either in radians or in degrees. The following equation is used to define a plane angle:

$$\varphi = s/R \,, \qquad (4.3)$$

where s is a circle arc length; R – circle radius length. What is not taken into account is the fact that the plane angle is a geometric figure whose definition says nothing about the circle. That's why the plane angle should not be described with the equation (4.3); in the same way, it shouldn't be described with trigonometric functions of the triangle's angles.

Rotation angle is a physical quantity describing rotation of a ray starting from the body's center of rotation against another ray that is considered as stationary. As a physical quantity the rotation angle is not defined by any quantity equation.

The base quantities have no quantity equations by definition, that's why nothing prevents us from choosing the rotation angle (angular displacement) as a base quantity of the ISQ. The SI already has a unit for such a base quantity, and therefore it has a dimension, too. The suggestion to make radian a base SI unit was already expressed before [15, 7, 4].

The article [16] suggests including the rotation angle (angular displacement) into the base quantities of the ISQ with the symbol A (from the word ‘angle’). Naturally, the dimension of A should be included into dimension of every quantity, the quantity equation of which includes either the rotation angle or the angular displacement. This pertains to such geometric quantities as the area of a circle, the surface of a sphere, the volume of the sphere, etc., if these quantities are included into the quantity equations of physical quantities.

4.5. In which Units Should the Rotation Angle Be Measured?

As follows from the definition of the angle as a geometrical figure, the area of which is limited by two rays, its area is infinite. To assess value of an angle, we can use ratio of the assessed angle to the full plane angle, formed by full revolution of one ray against another one.

In the general case, relation of two infinite quantities is indeterminate. But the ratio of the infinite area of the assessed plane angle to the infinite area of the full plane angle is a finite quantity, since the values of the ray lengths cancel each other out. The ratio can range from 0 to 1, or from 0 to the *full plane angle*, the unit of which is called revolution. In practice the full plane angle is divided into 360 fractional units (angular degrees); it's in these units that the measurement standards of plane angles are produced. Angular degrees, minutes, and seconds are provided in every reference book. This means, in practice the *degree measure* of the plane angle is used.

In theoretical physics, the *radian measure* of plane angle is most widely used; radian is also a fraction of a revolution, though not sufficiently precise one. Moreover, radian doesn't have a measurement standard. Therefore it's quite puzzling that radian is chosen in the SI as a unit of angle, and the unit revelation is prescribed not to be used. If this is so, they should have also removed the definition of the plane angle as a geometrical figure from metrological standards, but the definition is still there. This is the first inconsistency in the definition of the angle. The second inconsistency is the fact that in every definition and metrological standards the plane angle is assessed as a ratio of angles, not a ratio of areas.

Also, if the plane angle is defined as a ratio of a circle arc to the radius of the same circle, then both the dividend and the divisor should be measured; in practice, neither of them is measured. The length of an arc in physics is not a number; it's a distance travelled by the body's center of mass, a physical dimensional quantity.

The author [4] writes that "it's possible to choose revelation as a base unit, but this would require revision of quantity equations for the coherent derived units of other angular quantities. The practical compromise can consist in the use of radian as the base unit of angle, so that angular velocities, for instance, will be measured in rad/s, not in the ambiguous s^{-1} that is in use now. The SI unit for the solid angle, steradian, will be defined as a special name for the unit rad^2." In our opinion, such revision of the plane angle unit would have one more positive outcome: it would make theoretical metrology closer to the measurement practice; in fact, angular velocities are already being measured in rad/s (rad s^{-1}), in atomic physics – in rev/s (rev s^{-1}), in technology – in rev/min (rev min^{-1}).

According to one encyclopedia, "there is no essential difference between the degree measure and the radian measure of a plane angle, but the use of the radian measure allows to simplify many formulas". This statement is not indisputable, since simplification of one thing leads to complication of another. The use of the factors 2π and 4π complicated writing of quantity equations for electromagnetic physical quantities so much, that rationalized systems of units were introduced.

Unfortunately, at the moment it is difficult to replace radians with revolutions in the standards pertaining to the plane angle units. We can temporarily accept radian to be used in cases when it's convenient – under one important condition: radian should be defined as a fraction of a revolution according to the equation 1 radian = $1/2\pi$ rev.

4.6. What Should Be the Units of Quantities Describing Rotation?

Let's show how the quantity equations, dimensions and units of the quantities describing rotation will look like, if the dimension of rotation angle A and the unit 'revolution' are used, as it is suggested in the article [16].

Kinematic quantities. The dimension of angular velocity $\boldsymbol{\omega}$ of a rotating body becomes AT^{-1} with the unit rev s^{-1}, and dimension of the angular acceleration $\boldsymbol{\varepsilon}$ of a rotating body becomes AT^{-2} with the unit rev s^{-2}.

A new quantity was recently introduced into the SI – angular frequency, also known as rotation frequency. In the Section 5.3 we have discussed the incorrectness of this concept and unsoundness of its introduction into physics.

Torque moment. It is defined from the equation of energy gain of a rotating body dW

$$dW = \mathbf{M}\, d\boldsymbol{\varphi}_{rot} \ , \qquad (4.4)$$

where **M** is the torque moment. As follows from the equation (4.4), the unit of the torque moment **M** has the dimension joule per revolution (joule rev^{-1}). In the SI this should correspond to the unit joule per radian (joule rad^{-1}).

Sometimes torque moment is defined from the time derived equation (4.4), that is, from the equation for instantaneous power P of a rotating body:

$$P = \mathbf{M}\, \boldsymbol{\omega}_{rot} \ . \qquad (4.5)$$

The definition of **M** from the equation (4.5) contradicts the causality principle.

The unit of the torque moment **M** in the SI now equals N m = joule; the unit rad^{-1} doesn't exist. Why the radian unit can be used to form the unit of angular velocity but cannot be used to form the unit of the torque moment? The SI has no answer for this question.

The body's moment of inertia. The Newton's second law for the rotational movement form should be written the following way:

$$\boldsymbol{\varepsilon}_{rot} = \mathbf{M}/ J_z \ , \qquad (4.6)$$

where $\boldsymbol{\varepsilon}_{rot}$ – angular acceleration of the rotating body; J_z – the body's moment of inertia. According to the equation (4.6) and using the unit joule rev^{-1} for the torque moment **M** we conclude that the unit of the moment of inertia J_z should equal J s^2 rev^{-2}, which corresponds to the unit J s^2 rad^{-2} in the SI. At present the moment of inertia J_z has the unit kg m^2 in the SI.

If we decode the unit kg with the help of the Newton's second law $\mathbf{F} = m_{in}\mathbf{a}$, where m_{in} – the body's inert mass, and use the equation kg = J s^2 m^{-2}, we can easily conclude that the unit of the moment of inertia kg m^2 equals the unit J s^2. If we now compare this unit with the obtained from the equation (4.6) unit J s^2 rad^{-2}, we can see that in the SI the unit of J_z has "lost" rad^2 in the divisor.

Angular momentum. It is defined with the equation:

$$\mathbf{L}_z = J_z\, \boldsymbol{\omega}_{rot} \quad . \qquad (4.7)$$

If we substitute the unit of J_z that equals J s^2 rev^{-2}, and the unit of $\boldsymbol{\omega}$ that equals rev s^{-1}, we will get the unit of the angular momentum $\mathbf{L}_z$ that equals J s rev^{-1}; in the SI this corresponds to J s rad^{-1}. This unit is different from the unit of the Planck constant h that equals J s $quantum^{-1}$, since the angular momentum describes continuous rotation, while the Planck constant is used in quantum mechanics.

Moment of momentum. In the literature the moment of momentum is often considered a synonym for the angular momentum; it's denoted with the same symbol $\mathbf{L}_z$. But the physical content of the moment of momentum is different, as well as its quantity equation:

$$\mathbf{L}_z = [\mathbf{R}\ \mathbf{p}] = [\mathbf{R}\ (m_{in}\, \mathbf{v}_\tau)]\ , \qquad (4.8)$$

where the tangential velocity of the body moving along the orbit $\mathbf{v}_\tau = [\mathbf{R}\ \boldsymbol{\omega}_{orb}]$. This means that angular velocities included in the equations (4.7) and (4.8) are different both in value and in content. That is why the angular moment and the moment of momentum should not be considered as synonyms. Using the unit of the radius of curvature **R** that equals m rev^{-1}, we obtain the unit of the moment of momentum that equals kg m rev^{-1} s^{-1}, or J s rev^{-1}.

The moment of momentum and the angular momentum have identical units, but they shouldn't be considered as synonyms. The content of the quantity is defined by its quantity equation, not its unit.

Moment of force. There is a difference between the moment of force and the torque moment, which is not noticed in modern metrology. The moment of force **N** is connected with the force with the following equation:

$$\mathbf{N} = [\mathbf{r}\ \mathbf{F}], \qquad (4.9)$$

where **r** is a radius vector traced to the origin of force **F**. According to the equation (4.9), the unit of the moment of force is N m. The coincidence of this unit with the unit of energy, joule, that is often discussed in articles by metrologists, is purely incidental, since the moment of force **N** is a supplementary *static* unit used for technical calculations. The moment of force **N** doesn't create rotation; it just characterizes the possibility of rotation to happen. Unlike the moment of force **N**, the torque moment **M** is a *dynamic* quantity, and its unit is J rev^{-1}. The units of the moment of force and the torque moment don't coincide.

If we identified the torque moment with the moment of force, that is, if we defined the torque moment with the equation

$$\mathbf{M} = [\mathbf{R}\ \mathbf{F}] \quad , \qquad (4.10)$$

where **R** – radius of curvature of the movement trajectory, we would get a different unit of force equals J m^{-1} rev^{-1} = N rev^{-1}, since the unit of the radius of curvature **R** is m rev^{-1}, as it was shown in the Section 4.3. (In the SI this would correspond to the unit J m^{-1} rad^{-1} = N rad^{-1}). But the unit of force equals J m^{-1}. This is one more reason not to identify the torque moment with the moment of force.

If we still tried to identify them, there would be necessity to distinguish the unit of force **F** in the straight-line movement form from the unit of the *rotary force* $\mathbf{F}_{orb}$ in the orbital movement form.

The reasoning explained in this Section shows that there is no consensus concerning the rotation angle quantity, and that in the SI the unit of the rotation angle is sometimes applied in the units of rotational quantities, and sometimes not. In this regard the author thinks that his opinion once shared in the article [7] – "it's time to end the ostrich policy", is still relevant.

5. Units of the Quantities of Periodic Processes

In modern metrology and terminology of periodic processes there are examples of terms and units, which raise questions with no answers. For instance, why does SI have three ways to represent oscillation phase, each one leading to a different equation to define oscillation frequency and a different frequency unit, though their dimension is the same? How should one understand the 'angular frequency' term in oscillations with no angular movement? Why does SI suggest to measure an oscillation period in seconds, though second measures duration of the period and not the period itself? Why is wavelength measured in meters, and not in meters per one wave? Let's try to answer these and other similar questions.

5.1. Dimensions and Units of Oscillation Phase and Oscillation Frequency

There are three ways to represent oscillation phase and oscillation frequency in the SI. Oscillation frequency, while having the same dimension, has three different quantity equations and three different units in the SI. This diversity is the result of the fact that in the first case the oscillation phase is an *integer number of periods N* within the time interval Δt, in the second case it is *one period* with duration *T*, and in the third case it is expression ($\omega_0 t + \varphi_0$), which is a *no integer number*, where ω_0 is called angular frequency, and φ_0 – the initial phase, which describes fraction of the period. The term *cyclic frequency* is not recommended for use in the SI, though any oscillations are a cyclic process.

Table 1. Different Ways to Represent Oscillation Phase and Frequency in the SI

Option #	Oscillation phase				Oscillation frequency			
	Term	Symbol	Dimension	Unit	Term	Equation	Dimension	Unit
1	Number of periods	N	–	–	Oscillation frequency	$N / \Delta t$	T^{-1}	s^{-1}
2	Period duration	T	T	s	Oscillation frequency	$f = T^{-1}$	T^{-1}	Hz
3	Oscillation phase	$\omega_0 t + \varphi_0$	–	rad	Angular frequency	$\omega_0 = \mathrm{d}\varphi/\mathrm{d}t$	T^{-1}	rad s^{-1}

Option 1. Oscillations are considered as a *quantizable process*. This option takes into account only the time interval Δt, within which an integer number *N* of full periods take place. Oscillation frequency is expressed as ratio of the *number of periods* to the time interval Δt.

Let's pay special attention to the fact that the time interval Δt pertains not to the *oscillation period,* but to the *period duration*, for which the dimension T and unit second are natural. Thus there is no logic in reduction of the term 'period duration' to the word 'period'. It is written everywhere, that oscillation period is measured in seconds, though it's a gross error. Oscillation period is an object of cyclic process, an independent physical quantity. This is why it is so difficult for one's consciousness to realize the fact that the true unit of oscillation period is a piece, while second is the unit of period duration. It is difficult, but utterly necessary.

Option 2. The 'number of periods' term is not used in this option. Period is described as a time interval within which the phase changes by 2π. As a rule, the unit of oscillation frequency is written as hertz (Hz).

Oscillation frequency is defined as a quantity inverse to the period duration. In such definition of oscillation frequency there is no physical sense of frequency itself. The only thing that's left is a verbal formulation of the mathematical operation.

Option 3. It is based on the method of vector diagrams, in which the full oscillation phase includes an integer number of circles circumscribed by the radius vector on the orthogonal coordinate plane. This is where another term comes from – *circular frequency;* it is also not recommended for use in the SI.

The method of vector diagrams is a mathematical interpretation of the harmonic oscillations. It uses uniform (mental) rotation of the radius vector on the orthogonal coordinate plane. The value of radius vector corresponds to the value of oscillation amplitude, while the oscillation phase is explained as rotation angle of the radius vector. Projection of the end of the radius vector on the coordinate axis performs linear movement according to the following equation:

$$x = A\cos(\omega_0 t + \varphi_0), \qquad (5.1)$$

where x is the current value of the oscillating quantity; A – oscillation amplitude; $(\omega_0 t + \varphi_0)$ – *oscillation phase*; ω_0 – *angular frequency*; φ_0 – *initial phase*. Unlike the 1^{st} option, this 3^{rd} option considers oscillations as a *continuous cyclic process*.

If we disregard the 2^{nd} option, which pays no attention to the number of periods, we will have to consider the 1^{st} and the 3^{rd} options: the 1^{st} keeps the physical content of the oscillation process but pays no attention to oscillation phase, while the 3^{rd} pays attention to the phase, but interprets the physical content of the oscillation process with a mathematical abstraction.

5.2. On Confusion in Symbols of Oscillation Frequency

Writing of the equation (5.1) differs from the one given in reference books and textbooks with a lower index '$_0$'. This index is introduced on purpose, to show difference between physical and mathematical quantities: angular velocity of a rotating body ω from angular frequency ω_0; rotation angle of a rotating body φ from the rotation angle of radius vector φ_0. That's because oscillations can be of any nature, including those without any rotating movement.

Is it normal that one and the same symbol ω is used in literature to denote angular frequency, angular velocity of a rotating body, oscillation frequency of electromagnetic radiation, and frequency of alternating current in electric engineering? Note that these physical quantities are the ones in quite frequent use. Moreover, such confusion can be found in one and the same textbook, sometimes on the neighboring pages. In such cases the quantities that are different in content should have different symbols – or at least different indices. For this same reason, the equation (5.1) in this article is written against the standard, with lower indices "$_0$".

Here is one convincing example. In the electric technology the reactive resistance in AC circuits is often written as $X_C = 1/(\omega C)$ and $X_L = \omega L$, where C – the circuit's capacity, and L – inductance. But in fact there is no angular velocity ω in an AC circuit; instead, there is AC frequency f. Consequently, the reactive resistances should be written as $X_C = 1/(2\pi fC)$ and $X_L = 2\pi fL$. This is even more important when teaching about electric machines, where we often deal with armature's angular velocity ω, which is very different from AC frequency both in numbers and in content.

5.3. Incorrectness of the 'Angular Frequency' Term

Let's analyze the "angular frequency" concept (synonyms: radial frequency, cyclic frequency, circular frequency), defined as a scalar physical quantity, frequency measure of a rotational or oscillatory movement.

Indeed, both of these kinds of movement are cyclic, but their nature is essentially different. Oscillations are *multidirectional movement* that is not necessarily rotational, while rotation is a *unidirectional movement* described by a vector quantity, which is called angular velocity.

The reason of such incorrect combination of the two kinds of movement is the wide use of the method of vector diagrams. However, it makes one forget that the rotating radius vector on the coordinate plane in an abstract mathematical quantity, not a physical one. But even in this case we should speak of the angular velocity of the radius vector, and not the angular frequency.

The author has happened to see a scientific article with the following argument aimed to justify such the ambiguity of terminology: *"Everybody understands everything anyway"*. It is likely that everyone just got used to the status quo. And once one tries to consider it carefully, the status quo becomes hardly understandable, especially for students.

5.4. The Necessary Elaborations in Metrology of the Oscillatory Processes

Oscillation process is described by five physical quantities: amplitude, frequency, phase, number of periods, and period duration. Only one of them – period duration – has a concrete dimension, dimension of time. The dimension of amplitude is identical to the dimension of the oscillating quantity. As for the dimensions of the oscillation phase and the number of periods, modern metrology says nothing concrete.

Let's consider two variants of representing the oscillation process.

1. Continuous oscillation process, which corresponds a real process, in which the oscillation phase takes into account the fraction of the oscillation period. In the method of vector diagrams the phase is defined by the rotation angle of the radius vector. The rotation angle has no dimension in the SI; it has only a unit, radian. That is why in the Table 1 the dimensions of angular frequency, angular velocity, and oscillation frequency are the same and equal T^{-1}. The difference between these quantities in the SI can be seen only from their units, and even this is not always possible.

This difference will become clear if the rotation angle becomes a base quantity and gets its own unit [16, 4]. The detailed substantiation of such a possibility is given in the article [17] and briefly explained in Section 4 of this article. It is suggested to use symbol A as a dimension of rotation angle, and it is proven that its unit should be revolution, not radian. In Section 3 it is advised to use symbol N and unit 'piece' for another base quantity – the number of entities. While describing an oscillation process of this quantity can be called a 'period'. This will make the difference between the dimension of the angular velocity AT^{-1} with the unit 'revolutions per second' and the dimension of oscillation frequency NT^{-1} with the unit 'periods per second' more salient. It is logical to consider the unit 'hertz' (Hz) equal to the unit 'periods per second'.

2. Quantizable oscillation process. When it is being used, the number of periods is considered only as an integer number, and the oscillation phase is not discussed. The Table 2 shows how the dimensions and units will change if the suggested update of dimensions and units takes place.

One and the same dimension should not belong to quantities that are different in content. When periodic processes are considered in the SI, this condition is not observed, and if the suggested update takes place – as one can see from the Table 2 – it will be observed.

Table 2. Suggested Changes of Dimensions and Units

Term	Expression	Dimension	Unit	Term	Expression	Dimension	Unit
Periods (SI)				**Oscillation frequency (SI)**			
Number of periods	$\omega_0 t$	–	–	Oscillation frequency	$f_0 = \omega_0 t/\Delta t$	T^{-1}	s^{-1}
Period	T	T	s		$f_0 = T^{-1}$	T^{-1}	s^{-1}
Periods (SI update)				**Oscillation frequency (SI update)**			
Number of periods	N	N	per	Oscillation frequency	$f_0 = N/\Delta t$	NT^{-1}	per s^{-1}
Period duration	$T = \Delta t/N$	$N^{-1}T$	s per^{-1}				

5.5. The Necessary Elaborations in Metrology of the Wave Movement

To describe wave movement, along with the quantities describing oscillation processes, two other quantities are used: the *wave vector* **k** and the *wave number* $k = 2\pi/\lambda$, which is the module of the wave vector, where λ – *wavelength* that is measured in meters in the SI. The unit of the wave number in the SI is the so-called inverse meter (m^{-1}), the unit that doesn't make more sense than inverse second or inverse mole.

From the formula defining wavelength of electromagnetic radiation $\lambda = 2\pi c/\omega_0$ follows another formula for calculation of the wave number $k = \omega_0 /c$, where c – phase velocity of electromagnetic waves. When the method of vector diagrams is used with consideration of the dimension of the angular velocity of the radius vector ω_0 that equals AT^{-1}, and dimension of c that is equal to LT^{-1}, after analysis of dimensions we find dimension k, that equals AL^{-1} and has the unit rev m^{-1}. This corresponds to the unit rad m^{-1} in the SI. But it is definitely not inverse meter.

The same unit rad m^{-1} can be found in the equation for traveling wavefront displacement, $\xi = A \cos(\omega_0 t - kx + \alpha)$, where α – initial phase. In this case all the summands of the argument of the trigonometric function have the same unit, radian. In the SI, where the unit of k equals m^{-1}, the summand kx has no unit, while the summands $\omega_0 t$ and α have their units.

Each wave is in essence an entity of wave movement. That is why nothing prevents us from counting the number of waves as the number of entities of wave movement with dimension N. In this case the dimension of wavelength equals LN^{-1}, and its unit is m $wave^{-1}$. This makes the physical content of wavelength very clear: it is length of a single wave.

In physics, it is often convenient to use the quantum wave number $k_n = N/\lambda_n$, where N – number of quanta, and λ_n – wavelength corresponding the kind of radiation that is being considered. The quantum wave number k_n can have dimension $L^{-1}N$ and the unit quanta per meter (quanta m^{-1}), and wavelength λ_n – dimension LN^{-1} and the unit m $quantum^{-1}$. In this case k_n should be understood as the number of quanta per one meter.

5.6. On the Name of the Unit for Cyclic Quantities

The author of article [8] suggested to use symbol I and name *heis* as a unit for quantity with dimension one, that is a *no integer number*. For this case he considered using prefixes mega-, kilo-, santi-, milli-, micro-, pico-, etc. with the unit heis. Later in a different article [18] it was suggested to give this unit another name – uno, another symbol – U, and use this unit with prefixes for decimal fractions, percent, and ppm. The question of naming this unit is still open for discussions.

General Conclusions

Analysis of the "quantities with dimension one" has given the following results:

1. We have defined the components of this concept (similarity criteria, cyclic quantities describing rotation, oscillations and waves, and numbers of entities), and arranged them into a hierarchic diagram (Section 1.3).

2. We have shown that similarity criteria are ratios of dimensional quantities of the same kind, and therefore they can be described with the system or an off-system unit of the divisor of this ratio (Section 2).

3. The number of entities should be included into the ISQ as a base quantity with its own dimension and unit; the unit for the number of entities is called different names in different chapters of physics (Section 3).

4. The unit for the number of entities should be included into the unit of the Planck constant (Section 3.6).

5. The amount of substance $n = N/N_A$, where N_A – the Avogadro constant with unit mol^{-1}, should be excluded from the set of base quantities of the ISQ; instead, the similarity criterion $n_A = N/A_N$ should be used, where A_N – the Avogadro number (Section 3.2).

6. We provided classification of the forms of body movement, according to which the angle of the body's spinning rotation and angular movement of the center of mass of the body moving along a curved trajectory are different physical quantities assessed with the unit of plane angle (Section 4.1).

7. The angular quantities should be included into the ISQ as a base quantity with its own dimension and unit; their unit should be described by revolution, assessed with a full plane angle; the radian measure of an angle can remain as auxiliary and be used for theoretical calculations (Section 4).

8. The unit for the radius of trajectory's curvature should be meter rev^{-1}, not meter; the unit for the trajectory's curvature is rev m^{-1}, not m^{-1} (Section 4.3).

9. The unit for angular quantity should be present in the units of every quantity describing rotational movement, including the torque moment, the body's moment of inertia, the angular momentum, and the moment of momentum (Section 4.6).

10. The number of oscillation periods and number of waves are particular cases of the base quantity "number of entities" and therefore have their dimension and unit that should be present in dimensions and units of the oscillation phase, the oscillation frequency, and the wave number (Section 5).

11. The unit second measures duration of the oscillation period, not the period itself; oscillation frequency is measured in per s^{-1}, not in s^{-1}; wavelength is measured in m wave^{-1}, not in meters (Sections 5.1 and 5.5).

12. The units m^{-1} (inverse meter), s^{-1} (inverse second), mol^{-1} (inverse mole) and similar ones should be discarded, since they have no physical content.

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