Considerations on CODATA-evaluated values for the fundamental constants

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Introduction

Since 1969 a CODATA Task Group [1] is performing a valuable check of consistency of a large set of fundamental constants—recently re-named reference constants—by means of the so-called Least Squares Adjustment (LSA). At regular intervals, these evaluations produce a set of adjusted values and of the corresponding associated uncertainties.

Since most of these constants are multidimensional, this set of values also present a tool to check the degree of consistency of the set of the SI units concerned. Therefore, this tool is a complement to the great scientific value, also metrological, of the continuing measurement effort for decreasing the uncertainty in the value of the constants.

In recent times, a proposal has been brought forward by a group of metrologists to use some of these constants (c_0, h, e, k_B, N_A) as the bases for the re-definition of some of the base units of the SI [2], by stipulating their values[1]. This proposal was presented to the 2011 meeting of the CGPM, which only decided to "to take note" of the proposal in its Resolution 1 [3].

It is not the aim of this Note to discuss the validity of the proposal. The present aim is discussing the conceptual and practical differences between considering, on the one side, the experimental data on which the CODATA values are based, and, on the other side, the use of the adjusted CODATA values resulting from a typical CODATA evaluation.

The bases of CODATA evaluations

The bases of the CODATA evaluation can be found on the comprehensive publication [4–10] that follows each new release of a set of adjusted values and associated uncertainties of the constants. First, what is relevant in this Note is summarised from the CODATA publications, for each of the above constants and, in addition, for the molar gas constant $R = k_B \cdot N_A$ and for the fine-structure constant α that are also directly involved.

Speed of light in vacuum, c_0

Early determinations of c_0 were: 299 792(4.5) km s^{-1} (Essen, Gordon-Smith, 1948); 299 794.2(2.8) km s^{-1} (Aslakson, 1949); 299 792.5(1.5) km s^{-1} (Essen, 1950); 299 792.50(0.10) km s^{-1} (Froome, 1958); 299 792.56(0.11) (Simkin *et al*. 1967). The CODATA 1969 value was 2.997 9250(10)·10^{8} m s^{-1}.
The 1977 CODATA publication, justifying the 1973 adjustment of c_0, indicates that the value is based on a *single* experimental determination, that of Evenson *et al*. [11] of $\nu(CH_4)$: "The frequency and wavelength of the methane-stabilized laser at 3.39 μm were directly measured against the respective primary standards. With infrared frequency synthesis techniques, we obtain ν = 88.376 181 627(50) THz. With frequency-controlled interferometry, we find λ = 3.392 231 376(12) μm. Multiplication yields the speed of light c_0 = 299 792 456.2(1.1) m/sec, in agreement with and 100 times less uncertain than the previously accepted value. The main limitation is asymmetry in the

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1 In the following the term "stipulation" is used, often used instead and with the same meaning of "defined".

krypton 6057-Å line defining the meter”. Using the value of $\lambda(CH_4)$ recommended by CCT in 1973 based on three determinations whose accuracy was limited by the definition of the metre based on the Kr wavelength, the resulting value is instead c_0 = 299 792 458.33 m s^{-1} with an experimental standard deviation of 0.6 m s^{-1}, according to CODATA. [2] The 1973 adjustment preserved the latter value (“Without intending to prejudge any future redefinition of the metre or the second, the CCDM suggested that any such redefinitions should attempt to retain this value provided that the data upon which it is based are not subsequently proved to be in error” [5]), but not the uncertainty, set to 1.2 m s^{-1} ($4{\cdot}10^{-9}$ relative). The value, and its uncertainty, used instead for the 1973 LSA, as said in [5], was the Evenson one [11], (299 792 456.2 ± 1.1) m s^{-1} ($3.5{\cdot}10^{-9}$ relative). Later, Mulligan [12] in 1976 obtained 299 792 459.0(0.8) m s^{-1}.
The value 299 792 458 m s^{-1} was *stipulated* by the CGCM in 1983. Figure 1 summarises the changes in time of the value of c_0 and of the associated uncertainties.

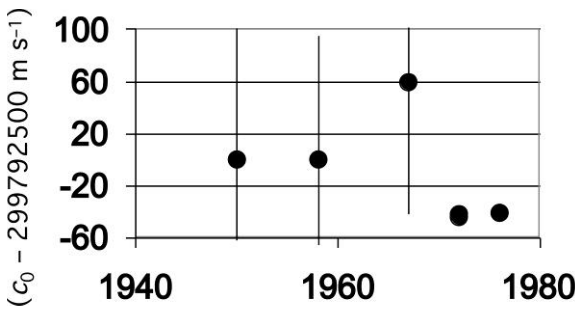


Figure 1. Experimental values of c_0 (difference with respect to the stipulated value)[3].

Planck constant, h

The 2010 CODATA[4] adjustment of h was basically based on two sets of new determinations.
The first was one bringing to the new adjusted value for N_A “A new value of the Avogadro constant N_A with a relative uncertainty of 3.0×10^{-8} obtained from highly enriched silicon with amount of substance fraction $x(^{28}Si) \approx 0.999\ 96$ replaces the 2006 value based on natural silicon and provides an inferred value of h with essentially the same uncertainty. This uncertainty is somewhat smaller than $3.6{\times}10^{-8}$, the uncertainty of the most accurate directly measured watt-balance value of h. Because the two values disagree, the uncertainties used for them in the adjustment were increased by a factor of two to reduce the inconsistency to an acceptable level; hence the relative uncertainties of the recommended values of h and N_A are 4.4×10^{-8}, only slightly smaller than the uncertainties of the corresponding 2006 values” [10]. The value is $6.626\ 070\ 09(20){\cdot}10^{-34}$ J s ($3.0{\cdot}10^{-8}$ relative)

The second method for determining the value of h experimentally was the “watt balance”, using several setups, the last being: NPL (completed in 2010 but published only in 2012), 6.626 071

[2] Oddily enough, pasting the value copied from the PDF file of the publication, which reads 299 792 458.33 m s^{-1}, one gets the value 299 792 459.33 m s^{-1}!
[3] When no uncertainty bar is shown, it means that it is shorted than the symbol size.
[4] Closing date of the 2010 adjustment, 2010-31-12: “A significant number of new results became available for consideration, both experimental and theoretical, from 1 January 2007, after the closing date of the 2006 adjustment, to 31 December 2010, the closing date of the current adjustment. Data that affect the determination of the fine-structure constant α, Planck constant h, molar gas constant R, Newtonian constant of gravitation G, Rydberg constant R_∞, and rms proton charge radius r_p are the focus of this brief overview, because of their inherent importance and, in the case of α, h, and R, their impact on the determination of the values of many other constants” [10].

23(133) $\cdot 10^{-34}$ J s ($2.0 \cdot 10^{-7}$ relative); METAS (completed in 2010 but published in 2011), 6.626 0691(20)$\cdot 10^{-34}$ J s ($2.9 \cdot 10^{-7}$ relative).

The 2010 CODATA adjusted value and uncertainty, after the LSA, is 6.626 069 57(29)$\cdot 10^{-34}$ J s (4.4 10^{-8} relative)—not including the above 2011 and 2012 values **[**13-14**]**. The previous 2006 CODATA adjusted value and uncertainty was 6.626 068 96(33)$\cdot 10^{-34}$ J s (5.0 10^{-8} relative).
All determinations of h are measures of a ratio: (h/m(e), h/m(n), h/m(Cs), h/m(Rb), $h/\Delta m(^{29}\mathrm{Si})$, $h/\Delta m(^{33}\mathrm{Si})$, all depending on the value of N_A; $h/2e$, h/m(K), h/e.

Subsequent the closing of the CODATA 2010 adjustment [2], further values were supplied. They came from the NPL watt balance moved to NRC: after a systematic error was detected in early 2012 by Robinson [13], the results published by Steele *et al.* in 2012 [14] provided the value h(watt balance) = 6.626 070 63(43)$\cdot 10^{-34}$ J s. In the mean time IRMM [15], revising its determination of isotopic composition of ^{nat}Si, provided the value: 6.626 0674(22)$\cdot 10^{-34}$ J s. The value h(NRC-11 ^{28}Si) = 6.626 070 55(21)$\cdot 10^{-34}$ J s also reported in [14] is *not* considered here, because CODATA adjusted values of other constants are said to have been used for its computation.

Two further 2012 papers [16, 17] attempt to obtain from the set of experimental data from 1979 to 2012, including the last NRC one, a 'consensus' value for h. The first is based on a larger set of data (Fig. 2c) with a weighted mean h_0 = 6.626 070 07(29)$\cdot 10^{-34}$ J s, and brings to four additional estimates, all with a value of h lower than h_0 and a larger uncertainty, up to $\Delta h/h_0 = -3(7)\cdot 10^{-7}$ J s. The second, using a smaller set of data (Fig. 2a plus the newest NRC), brings to four estimates between h = 6.626 0696(6)$\cdot 10^{-34}$ J s and h = 6.626 0697(2)$\cdot 10^{-34}$ J s, closer to the CODATA 2010 adjusted value.
Figure 2 summarises the changes in time of the value of h and of the associated uncertainties—see [10] and [16] for data identification.

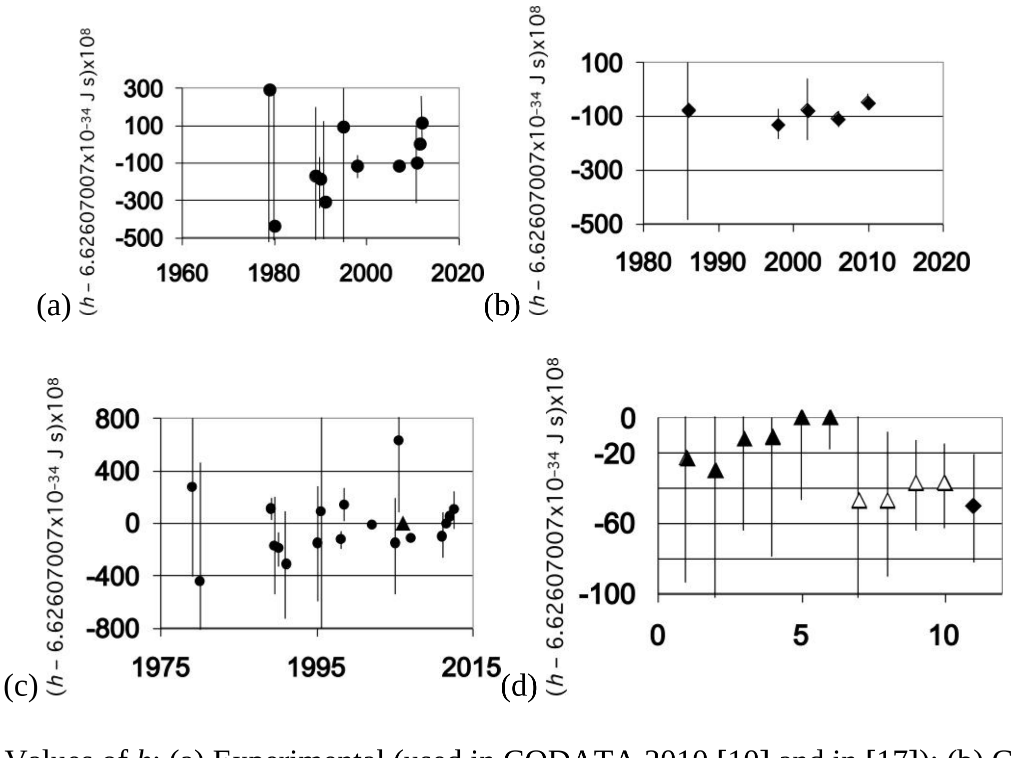


Figure 2. Values of h: (a) Experimental (used in CODATA 2010 [10] and in [17]); (b) CODATA 1986–2010; (c) Experimental from [16]: triangle CODATA 2006; (d) Mean-value 2012 evaluations: open [17], filled [16], diamond CODATA 2010.

Elementary charge of the electron, e

The 2010 CODATA adjustment of e is *not* based on direct experimental data, In fact, since the 1973 the CODATA adjustment [5] indicates: "since $(2e/h)_{BI69}$ is now an auxiliary constant, the elementary charge, e, can be expressed in terms of the new variables and $(2e/h)_{BI69}$:

$$e = (\alpha K(\Omega_{BI69}/\Omega))/c_0(\mu_0/4)(2e/h)_{BI69}\ {}^{5} \tag{29.11}$$

Thus it is no longer stochastically independent of the other adjustable variables and may be eliminated from the least-squares solution". More recently, the value of e is obtained from:

$$e^2 = 2h\alpha/\mu_0 c_0$$

where α is the fine-structure constant.

The 2010 CODATA adjusted value and uncertainty, after the LSA, is $1.602\ 176\ 565(35)\cdot10^{-19}$ C (relative uncertainty $2.2\ 10^{-8}$). The previous 2006 CODATA adjusted value and uncertainty was $1.602\ 176\ 487(40)\cdot10^{-19}$ C ($2.5\cdot10^{-8}$ relative). The 2002 CODATA adjusted value and uncertainty was $1.602\ 176\ 53(14)\cdot10^{-19}$ C ($8.5\cdot10^{-8}$ relative). The 1998 CODATA adjusted value and uncertainty was $1.602\ 176\ 462(63)\cdot10^{-19}$ C ($3.9\cdot10^{-8}$ relative). The 1986 CODATA adjusted value and uncertainty was $1.602\ 177\ 33(49)\cdot10^{-19}$ C ($30\cdot10^{-8}$ relative).

Figure 3 summarises the changes in time of the value of e and of the associated uncertainties.

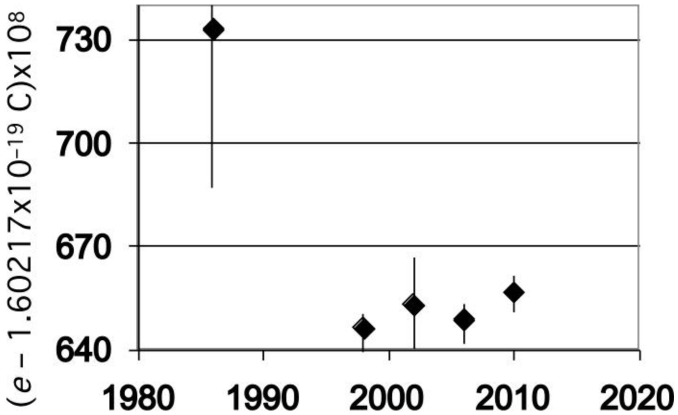


Figure 3. CODATA values of e (1986–2010).

Boltzmann constant, k_B

The 2010 CODATA adjustment of k_B was based on one new direct experimental determinations of the Boltzmann constant [18], and on other only indirect measurements: one based on the new adjusted values of the Planck constant h (see above), k_B/h [19], or one based on the new adjusted values of the molar gas constant R and N_A (see below), $k_B = R/N_A$.

The 2010 CODATA adjusted value and uncertainty, after the LSA, is $1.380\ 6488(13)\cdot10^{-23}$ J K^{-1} ($0.91\cdot10^{-6}$ relative). The previous 2006 CODATA value and uncertainty (still based on previous indirect determinations used for the 2002 and 1998 ones) was $1.380\ 6504(24)\cdot10^{-23}$ J K^{-1} ($1.7\cdot10^{-6}$

[5] In (29.11) the speed of light c_0 is originally written c, since in 1963 it was not yet a stipulated number.

relative). The immediately previous 1986 CODATA adjustment provided the value 1.380 658(12)·10^{-23} J K^{-1} (8.5·10^{-6} relative) and the 1973 CODATA adjustment the value 1.380 662(44)·10^{-23} J K^{-1} (32·10^{-6} relative).

Before the direct NIST 2007 value, no direct determinations of k_B were ever available, but the value was derived from $k_B = R/N_A$.

On the other hand, after the closing date for the 2010 CODATA adjustment, one paper has been published in 2012 on k_B [20]. Obtained with the dielectric constant gas thermometer, it provided two values, for measurements at low temperature (LT) and at the triple point of water (TPW), respectively: $k_{B,LT}$ = 1.380 657(22)·10^{-23} J K^{-1} and $k_{B,TPW}$ = 1.380 654(13)·10^{-23} J K^{-1}. Thus the mean value of all the experimental values is 1.380 654(7) J K^{-1} (5.4·10^{-6} relative), *not* compatible with the 2010 CODATA adjusted value.

Figure 4 summarises the changes in time of the value of k_B and of the associated uncertainties.

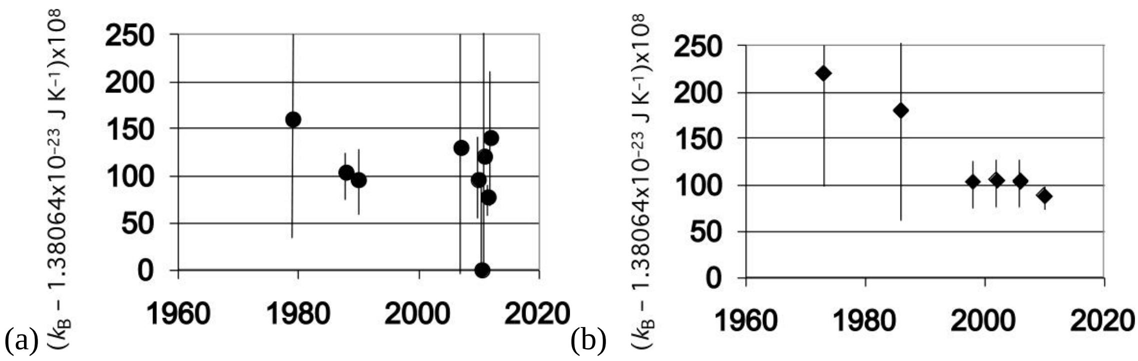


Figure 4. Values of k_B: (a) Experimental; (b) CODATA 1986–2010.

Avogadro number, N_A

The 2010 CODATA adjustment of $N_A = M_u/m_u$, i.e. the ratio of the molar mass and atomic mass constants, was based on the latest measurements on enriched silicon.

The 2010 CODATA adjusted value and uncertainty, after the LSA, is 6.022 141 29(27)·10^{23} mol^{-1} (4.4·10^{-8} relative): "the uncertainties used for them [N_A and h] in the adjustment were increased by a factor of two to reduce the inconsistency to an acceptable level; hence the relative uncertainties of the recommended values of h and N_A are 4.4·10^{-8}, only slightly smaller than the uncertainties of the corresponding 2006 values" [10]. This uncertainty enlargement is now *no more valid*, after the 2011-12 new results available for the watt balance experiments (see above Planck constant h).

The best experimental value obtained from enriched silicon was 6.022 140 82(18) ·10^{23} mol^{-1} (relative uncertainty 3.0 × 10^{-8}) [21]. The previous 2006 CODATA adjusted value and uncertainty was 6.022 141 79(30) ·10^{23} mol^{-1} (relative uncertainty 5.0 × 10^{-8}), based on natural silicon [9].

Figure 5 summarises the changes in time of the value of N_A and of the associated uncertainties.

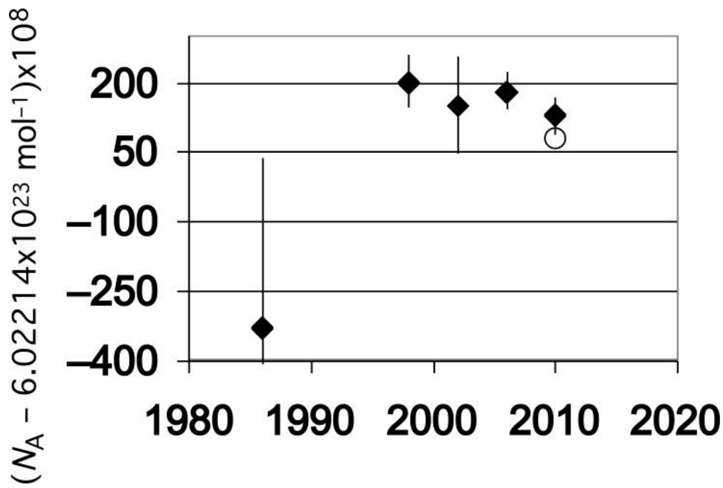


Figure 5. Values of N_A: CODATA 1986–2010: the open circle is the experimental value for enriched silicon.

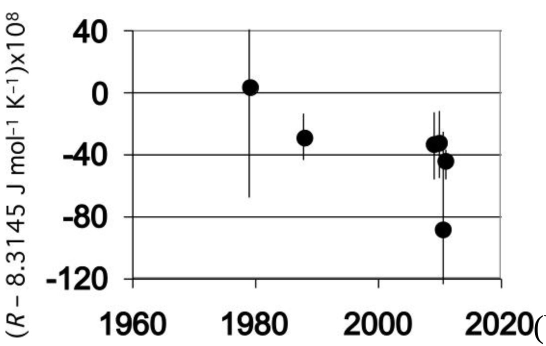
Molar gas constant, R

The 2010 CODATA adjustment of R is based on six direct experimental determinations [10]. The mean value of these determinations is R = 8.314 463(30) J mol^{-1} K^{-1} (3.6·10^{-6} relative). The first determination considered is quite older than the others: the mean value obtained from these data by omitting it, becomes R = 8.314 455(25) J mol^{-1} K^{-1} (3.0·10^{-6} relative).

The 2010 CODATA adjusted value and uncertainty, is 8.314 4621(76) J mol^{-1} K^{-1} (0.91·10^{-6} relative): note the additional digit.

The previous 2006 CODATA adjusted value (identical to the 2002 and 1998 ones) and uncertainty (based on the two oldest experimental determinations only), was 8.314 472(15) (1.7·10^{-6} relative). This should be compared with the mean value of these measurements 8.314 488(23) J mol^{-1} K^{-1} (2.8·10^{-6} relative). The 1986 CODATA adjusted value and uncertainty was 8.314 510(70) J mol^{-1} K^{-1} (8.4·10^{-6} relative).

Figure 6 summarises the changes in time of the value of R and of the associated uncertainties.

(a) 

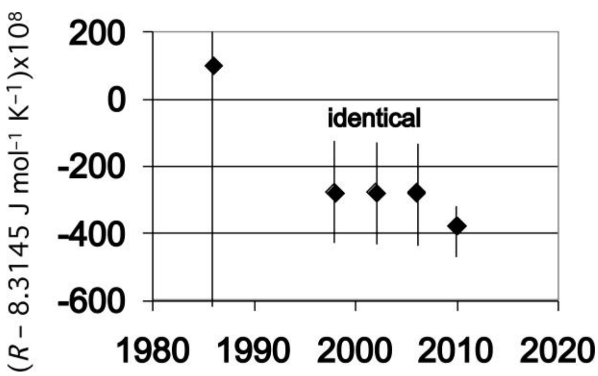
(b) 

Figure 6. Values of R: (a) Experimental; (b) CODATA 1986–2010.

Fine-structure constant, α

The CODATA adjustments of α are *not* based on direct measurements. The 2010 adjustment was based on 12 *indirect* experimental determinations with sufficiently low uncertainty [10]. Actually, α is computed from either: the electron magnetic moment anomaly, a_e; $h/m(^{87}\mathrm{Rb})$ or $h/m(^{133}\mathrm{Cs})$, where h is the Planck constant and m is the atomic mass of ^{87}Rb or of ^{133}Cs; the von Klitzing constant, R_K (where $R_K = h/e^2$); the experimental value of γ'_x in SI units, $\Gamma'_{P-90}(\mathrm{lo})$, obtained from a low-field experiment determining $\gamma'_x / K_J R_K$, where K_J is the Josephson constant; the hyperfine splitting of muonium, $\Delta\nu_{Mu}$; or, from ν_H, ν_D.

The 2010 CODATA adjusted value and uncertainty, is 7.297 352 5698(24)$\cdot 10^{-3}$ ($3.2 \cdot 10^{-10}$ relative). The previous CODATA adjustments were as follow: 7.297 352 538(50) $\cdot 10^{-3}$ (2006), 7.297 352 568(24) $\cdot 10^{-3}$ (2002), 7.297 352 533(27) $\cdot 10^{-3}$ (1998), 7.297 353 08(330) $\cdot 10^{-3}$ (1986).

Figure 7 summarises the changes in time of the value of α and of the associated uncertainties.

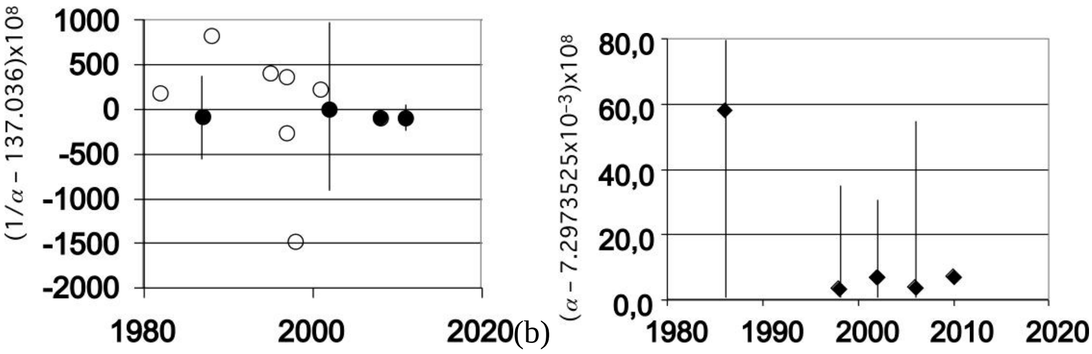
(a) 

Figure 7. Values of α: (a) Experimental $1/\alpha$ (the uncertainty of the open-circle data are larger than 3000); (b) CODATA 1986–2010.

The meaning of the CODATA adjusted values

The basic assumption underlying a LSA is that, by adjusting the values in a set of data by a carefully chosen process, it is possible to minimise the standard uncertainty of the set. The deviations from the measured values (called "adjustments") are considered as unknown variables to the set. The value of one of the data of the set, arbitrarily chosen, has to be kept fixed, i.e. the changes are all relatives to this datum. In the case of the CODATA, the magnetic constant $\mu_0 = 4\pi \cdot 10^{-7}$ H m^{-1} is set constant (stipulated). This generates the degree of freedom necessary to avoid the set of LS equations becoming undetermined and it means that all other CODATA values depend on this stipulated value.[6]

The adjustments of the other data can be non-significant if their values lye within a stated consistency level, typically the standard uncertainty, or can be significant if they exceed these limit, indicating an actual inconsistency between some pairs of measured values of the constants: for

[6] Because since 1983 the speed of light in vacuum c_0 is stipulated too, also the electric constant $\varepsilon_0 = 1/\mu_0 c_0^2$ is constant. Its value is calculated from the product of a stipulated value, c_0, and a real number, $\mu_0 = 4\pi \cdot 10^{-7}$, taken for exact. The value of ε_0, calculated instead from the value of c_0 before stipulation, would have a finite number of digits and an associated uncertainty. Its subsequent stipulation—rounding at the first uncertain digit (see later)—would bring, in this case, to a finite number of digits: 8.854 187 80$\cdot 10^{-12}$ F m^{-1}, different from the former (8.854 187 817. . . $\cdot 10^{-12}$ F m^{-1}).

example, this happened for the 2010 values of the Planck constant obtained though the Avogadro silicon project or through the watt-balance measurement procedure.

In all instances “adjustment” means that the values of the data are altered with respect to the initially measured values, i.e. to the experimental mean values originating from experimental data. The benefit consists in fact that the uncertainty attributed to the adjusted data is smaller, since the standard uncertainties in the adjusted set are smaller. They have been computed from a much higher number of data than is the case for each single constant.

The meaning of the LSA process is that, by suppressing the evaluated bias from each of the original data, the set is ‘compacted’, and an optimum (maximum) degree of compatibility within the set, i.e. of the data with each other, is attained.

What are the benefits?

The CODATA task is very valuable tool and a *unique* way to check the compatibility of the values assigned to each of them in a set of independent (in principle) values of constants and units, as the values for constants and units are supposed to be.

The addition, in the course of time, of new data enables evaluating the stability of the compatibility trend, and hopefully its improvement, with time.

Another benefit arises from the fact that several constants considered by CODATA are algebraic combinations of others. In this case, even the addition to the set of only a limited number of new data causes the recalculation of the whole set, and consequently the assignment of new adjusted values even for constants for which no new data have become available in the same period of time. In the case of an improvement in the knowledge of some constants, others will benefit of a reduced uncertainty after application of the LSA, associated with a new adjustment of their value. For example, this happened in 2010 for the Boltzmann constant, k_B, due to new measurements of the molar gas constant R.

What the LSA should *not* mean

The adjustment of the data values according to the LSA is justified as a minimisation of the set inconsistency in the sense of the LSA criterion, and, in addition, all adjusting terms, a_i, are undetermined by a constant value, a_0, identical for all original values of the set. The value of a_0 is set by assigning the value $a_r = 0$ to a constant arbitrarily chosen as reference.

For the constants whose values are experimentally determined, the LSA alters the original (mean) experimental value by the adjusting term, which, in turn, includes an arbitrarily-chosen contribution arising from the choice of the stipulated constants in the set.

As a consequence, the CODATA adjusted *values* of these constants *should not be conceptually confused* with the values obtained from the experimental determination(s) for the constants’ values, nor with the statistical evaluation of a summary parameter of them, like the mean, weighted mean, etc..

An example of a difference between the CODATA adjusted value and the measured mean value is the molar gas constant R.[7] The mean value of the experimental data used for the 2010 adjustment is 8.314 463(30) J mol^{-1} K^{-1} (or 8.314 455(25) J mol^{-1} K^{-1} if skipping the oldest value). The adjusted 2010 value is 8.314 4621(76) J mol^{-1} K^{-1}. Note the additional digit reported by CODATA.

Another example is the Avogadro constant, N_A.[7] The experimental 2010 value is 6.022 140 82(18)·10^{23} mol^{-1} while the CODATA 2010 adjusted value is 6.022 141 29(27)·10^{23} mol^{-1}, showing two barely overlapping uncertainty intervals.

A further example is the Boltzmann constant k_B.[7] The mean value of the two direct measurements is 1.380 654(7) J K^{-1} (5.4·10^{-6} relative), while the CODATA 2010 adjusted value is 1.380 6488(13)·10^{-23} J K^{-1} (0.91·10^{-6} relative), not compatible with the former.

Still another example is the decision that the CODATA had to take in 2010 about the treatment concerning the Planck constant, h,[7] being the two pivot values not consistent. As indicated in the relevant Section above, they had to increase the associated uncertainty and the value was barely consistent with that the CODATA 2006 one and is not consistent with h_0 [16], the weighted mean of the experimental data—though in Fig. 2d the weighted mean results to be the highest estimate in a set obtained by using different statistical methods of evaluation of the experimental results.

An example of adjusted values that are different in different adjustments *without being supported by direct experimental data* is the elementary charge e. Starting from the 1986 CODATA adjustment, until the 2010 one they were: 1.602 177 33(49)·10^{-19} C, 1.602 176 462(63)·10^{-19} C, 1.602 176 53(14)·10^{-19} C, 1.602 176 487(40)·10^{-19} C and 1.602 176 565(35)·10^{-19} C, respectively. Figure 3 shows its variations and their compatibility. Notice that, being based on α, whose value is based, in turn, on the measurement of other physical quantities, the value of e depends also on the value of h (through α), R_K , K_J, $\Delta\nu_{Mu}$, or from $m(^{87}Rb)$ or $m(^{133}Cs)$, depending on the source of the data.

As to the *uncertainty* assigned to the adjusted values, the LSA reduces, sometimes dramatically, the associated standard uncertainties, a direct benefit of the LS method arising from statistical features when a large set of values is involved (and showing remarkable consistency).

However, these *uncertainty* levels associated with the adjusted values of the constants *should not be conceptually confused* with the uncertainties achieved experimentally.

A clear example of this fact is the reduction in uncertainty of the Boltzmann constant, k_B, in the period 2006–2010. As seen above, the two CODATA adjustments brought to 1.380 6504(24)·10^{-23} J K^{-1} (1.7·10^{-6} relative) and 1.380 6488(13)·10^{-23} J K^{-1} (0.91·10^{-6} relative), respectively, though no new direct data for k_B were considered in 2010. The benefit resulted exclusively from an improvement in the knowledge of the molar gas constant R. The *minimum* experimental uncertainty claimed in the published measured values until 2010 adjustment is 1.5·10^{-6} relative, compared with the CODATA 0.91·10^{-6} relative.

Another example is the elementary charge e, for which the relative uncertainty from 1986 to 2010 improved dramatically as follows: 30·10^{-8}, 3.9·10^{-8}, 8.5·10^{-8}, 2.5·10^{-8}, 2.2·10^{-8}. No direct data are reported for this period: thus the fluctuations and the overall reduction of uncertainties (see Figure

[7] See relevant Section.

3) were exclusively generated by a better knowledge that was gained on other constants from which *e* is calculated.

Stipulation of uncertain values

Stipulating a measured value suppresses the associated measurement uncertainty datum, adding a potential problem to the use of adjusted values.

At present, this was only the case for c_0. The method followed by the CCDM for choosing the value, 299 792 458 m s^{-1}, indicates the correct road. But a note about the last digit is necessary, related to the issue of the uncertainty, since, once the decision is taken, valid forever, the stipulated value is exact by definition. The experimental value was 299 792 458.33 m s^{-1}, so the rounding to 8 is correct. However, the uncertainty, as already indicated above, was 0.6 m s^{-1} (or 1.2 m s^{-1} for the adjusted value). Therefore, the last digit 8 is in fact *uncertain*: it could have been 9 or 7. It seems contradicting the concept of stipulated value the use of an uncertain digit.
In the specific case, if the uncertain digit is omitted, the expression of the speed in km s^{-1} is no more possible: 299 792 46x. One possibility is to use 299 792.46 km s^{-1}, or to express it as in CODATA 1969, 2.997 9246·10^{8} m s^{-1}.

In general, the stipulated value being exact by definition, there is the problem to express it issuing only the correct number of digits (the use of all the digits of the CODATA *uncertain* value is not correct for the reasons indicated), normally expressed with two digits, the second having only the function of a 'guard digit'.

For example, let us consider the case of the experimental value of k_B, 1.380 653(13)·10^{-23} J K^{-1}: in this expression, the last digit, 3, is affected by an uncertainty and is reported only as a "guard digit", while the previous digit, 5, is affected by the significant uncertainty digit, 1. It seems contradictory to the meaning of "stipulation" to stipulate this experimental value as k_B = 1.380 653·10^{-23} J K^{-1} — thus exact by definition—since this would upgrade the "guard digit" to an exact digit.

Also, should the uncertainty having been expressed with only one digit, a correct expression of uncertainty too, the same value would have been written 1.380 65(1)·10^{-23} J K^{-1}. Let us now assume that this is the expression of a generic *uncertain* result: which should be the correct expression of the stipulated value? As already noted for the case of c_0, the last digit can be a 4 or a 6. The two limits of the uncertainty interval would require rounding of 6 to the lower digit, 5, or to the upper digit, 7, respectively. This may be the only good reason to keep the last digit 5 in the stipulated value: k_B = 1.380 65·10^{-23} J K^{-1}; otherwise, generally the stipulation should only involve the exact digits—i.e., unaffected by uncertainty.

Algebraic expressions of stipulated values

Stipulation of algebraic expressions is a frequent case found in CODATA documents. That is also the case for some of the constants included in the current proposal for the attention of the CGPM, with the obviously exception of c_0, already stipulated.

For the **Planck constant**, *h*, all determinations are measures of a ratio: h/m(e), h/m(n), h/m(Cs), h/m(Rb), $h/\Delta m(^{29}\mathrm{Si})$, $h/\Delta m(^{33}\mathrm{Si})$, all depending on the same value of N_A; $h/2e$, h/m(K), h/e.

For the **elementary charge of the electron**, e, no experimental determinations are available. CODATA uses $e^2 = 2h\alpha/\mu_0 c_0$, and therefore one has to assume that it is computed according to this quantity equation. While μ_0 and c_0 had their value stipulated, h is proposed to be stipulated at the same time as e, while α is not a stipulated constant.

For the **Boltzmann constant**, k_B, the only direct measurement used in the 2010 adjustment was NIST-07. The previous adjustments back to 1998, were only based on the expression $k_B = R/N_A$, where R is not a stipulated constant.

For the **Avogadro constant**, N_A, the experimental determinations on silicon, initially natural and eventually isotopically enriched, forms the bases for the value. No need for computations from other constants.

Consequently, for e, h and k_B the CODATA adjusted value derives from computation of the adjusted values of the other constants, α, c_0, μ_0, h and R, where μ_0 and c_0 are stipulated numbers.

For example, $k_B = R/N_A$. Thus, starting from the experimental values, $k_B = 6.022\ 140\ 82(18) \cdot 10^{23}$ / 8.314 463(30), the current proposal on the floor requires stipulating k_B and N_A, not R.

Should the stipulation of k_B be done at the same time of that of N_A, i.e. starting for both from the uncertain values, one gets $k_B = 1.380\ 649(30) \cdot 10^{-23}$ J K^{-1}, instead of the mean value computed from above, $1.380\ 654(16) \cdot 10^{-23}$ J K^{-1}. By rounding the first uncertain digit, the stipulated number becomes $k_B = 1.380\ 65 \cdot 10^{-23}$ J K^{-1}. This coincides with the stipulated value arising directly from the experimental values: $k_B = 1.380\ 65 \cdot 10^{-23}$ J K^{-1}.

For shake of comparison, with the use of the 2010 CODATA adjusted value, $1.380\ 6488(13) \cdot 10^{-23}$ J K^{-1}, one would get $k_B = 1.380\ 649 \cdot 10^{-23}$ J K^{-1}: note that the last digit is not experimentally justified.

Should instead the stipulation of k_B be performed after N_A having been stipulated, the stipulation of N_A would be 8.3145 mol^{-1} (the use of uncertain digits is not justified), whence $k_B = 1.380\ 655(30) \cdot 10^{-23}$ J K^{-1}. By rounding again at the first uncertain digit, the stipulated number becomes $1.380\ 66 \cdot 10^{-23}$ J K^{-1}, different from the previous stipulated value.

The case for the stipulation of e is similar, as it depends on α, h and from two already stipulated constants, μ_0 and c_0, but only h is proposed to be stipulated anew**.** In this case we cannot make a comparison with direct experimental determinations.
In the calculation that follows, the use of the CODATA 2010 value of α is first assumed, which is an adjusted one, but not the CODATA 2010 value of h ($6.626\ 069\ 57(29) \cdot 10^{-34}$ J s), because it is offset when using new data that became available after 2010 (see above the Section on h): the weighted average from [16] is used: $h_0 = 6.626\ 070\ 07(29) \cdot 10^{-34}$ J s.
Should the stipulation of e be done at the same time of that of h, i.e. starting for both from the uncertain values of h and α, one gets $e = 1.602\ 176\ 625 \cdot 10^{-19}(xx)\cdot 10^{-19}$ C. By rounding to the first *certain* digit (as already justified above), the stipulated number becomes $e = 1.602\ 1766 \cdot 10^{-19}$ C. Stipulated on the first *uncertain* digit, it would be $e = 1.602\ 176\ 63 \cdot 10^{-19}$ C.

The 2010 CODATA adjusted value is instead $1.602\ 176\ 565(35) \cdot 10^{-19}$ C: the corresponding stipulated value should be $e = 1.602\ 1766 \cdot 10^{-19}$ C. Stipulated on the first *uncertain* digit, it would be $e = 1.602\ 176\ 57 \cdot 10^{-19}$ C.

Should instead the stipulation of e be performed after that of h (the stipulation of h would be $6.626\ 070 \cdot 10^{-34}$ J s—not using of uncertain digits), one gets $e = 1.602\ 176\ 616$ (xx)$\cdot 10^{-19}$ C. By stipulating again to the first *certain* digit, the stipulated number becomes $1.602\ 1766 \cdot 10^{-19}$ C, identical to the previous stipulation; stipulated on the first *uncertain* digit, it would instead be different: $e = 1.602\ 176\ 62 \cdot 10^{-19}$ C.

Finally, using instead the CODATA 2010 value for h, in the first case $e = 1.602\ 176\ 564 \cdot 10^{-19}$ C, different from the previous case, but bringing to the same stipulated value using only *certain* digits. Stipulated on the first *uncertain* digit, it would instead be different: $e = 1.602\ 176\ 56 \cdot 10^{-19}$ C.

In the second case, stipulating first $h = 6.626\ 069\ 57(29) \cdot 10^{-34}$ J as $6.626\ 070 \cdot 10^{-34}$ J s, one gets again $e = 1.602\ 176\ 616\ (xx) \cdot 10^{-19}$ C, stipulated as $e = 1.602\ 1766 \cdot 10^{-19}$ C or $e = 1.602\ 176\ 62 \cdot 10^{-19}$ C, respectively.

In conclusion, in the four above cases, stipulating (incorrectly) to the first *uncertain* digit, one gets the following values: $e = 1.602\ 176\ 63 \cdot 10^{-19}$ C, $e = 1.602\ 176\ 57 \cdot 10^{-19}$ C, $e = 1.602\ 176\ 56 \cdot 10^{-19}$ C and $e = 1.602\ 176\ 62 \cdot 10^{-19}$ C, respectively.

Are algebraic expressions of stipulated values in turn stipulated values?

In [23, Table 2] the algebraic expressions of stipulated values, for example $R = k_B \cdot N_A$ or $F = N_A e$ (presently 96 485.3365(21) C mol^{-1}) or $K_J = 2e/h$ or $R_K = h/e^2$, are given for granted to become in turn stipulated values, i.e. also exact. This opinion is not a direct consequence of the issues illustrated in the previous Sections.

In fact, the stipulation of c_0, h, e, k_B, N_A is only a consequence of the fact that the definition of a unit cannot use an uncertain value (see, e.g., c_0, or the triple point of water, 273.16 K exactly, in the present definition of the kelvin).

Therefore, the exactness only applies for the purpose of the definition of those units. It does not suppress the uncertainty of the values of the constants for other purposes, like namely are the calculation of the value of another constant depending on one or more of the stipulated constants.

In other words, the stipulation of the value of a constant has not the general purpose of suppressing the uncertainty of the value existing before stipulation. Each of those constants should be specifically stipulated too, if needed; otherwise, they would retain the original uncertainty resulting from the uncertainties associated with the original experimental values of the involved constants, irrespective to the fact that their values have been stipulated in the definition of a measurement unit—in principle, ε_0 should be specifically stipulated too, see Footnote 5. Misunderstanding this issue might give rise to the very dangerous misunderstanding that the value is actually exact, and this not only among students or the general public.

One recent example. When at CERN some neutrinos were initially found to travel 'too fast', the correct question to rise would have been: 'are those neutrinos travelling faster than the defined value of c_0, or might the measurements on which the stipulated value of c_0 is based be incorrect?' Instead, no evidence exist that the physical problem placed by the measured speed value of the neutrinos was considered different from the theoretical one: 'how can neutrinos travel faster than light in vacuo?'. In fact, when later a missed systematic effect was found bringing the speed below the stipulated value of c_0, everything was considered resolved.

Conclusions

This paper has initially recalled how powerful is the tool used by CODATA in getting a measure of the consistency of the system of units, and how strong can be the decrease of the uncertainty associated to the adjusted values of some constants. The paper also shown that such a decrease for a constant can be obtained even without new measurements being available for that specific constant.

The paper has shown that there is a clear *conceptual* difference between the experimental values, based on which a mean value is assigned to a constant, and the CODATA adjusted value for the same constant: the purpose of the latter, by adjusting the experimental values, is to obtain the 'best measure' of the *consistency* of the set of constants.

As recalled in the paper, for a similar problem in the past, c_0, the preferred solution was to rely on the experimental values. A similar viewpoint was recently found in recent papers concerning h [16-17]. In addition, this paper has shown that for some of the constants involved, whose values are not (always) the direct result of measurements (h, e, k_B, N_A), there may be a problem in using one or more data arising from previous CODATA adjustments for the calculations.
The paper also has shown the problems that may arise from using uncertain digits for the stipulated values, that, in some cases, bring to different values depending on how the stipulation is performed.

For some of the constants included among the group considered by the recent proposal [2] for new definitions of some SI base units, these facts may bring to a different value when using one or the other as the stipulated value; and there are some of them, namely e, whose values (only) arise from calculations.

Finally, the paper has shown problems that may arise in calculating the values and associated uncertainties of constants related to others by algebraic expressions. In this respect, the opinion is expressed that the stipulation cannot automatically propagate to other constants.

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